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MATHEMATICS

JOHN R. CLARK, Chairman

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EDITORIAL

All persons who are cognizant of the great place which mathematics holds in the processes of civilization—war, bridge building, commerce, mining, philosophy, and pure science—are a bit dazzled by its almost supreme importance. Hence, they lend ready approval to its inclusion in the curricula of colleges, high schools, and even elementary schools. So nearly unanimous is this assent that mathematics teachers are sometimes impatient if skeptics raise questions regarding what mathematics and how much mathematics should be required for all pupils at each level of educational advance. Many mathematics teachers are indeed shocked if it is suggested that all mathematics beyond arithmetic computation be offered as electives or as parts of specific vocational or preprofessional curricula.

Nevertheless, these questions cannot be evaded; those who raise them cannot be finally bullied into silence by a dignified gesture regarding the noble history of mathematics as an expression of pure truth, or the great contribution that mathematics has made to world destruction through war or to world betterment through the arts of peace. It is all too obvious, moreover, that the majority of high-school mathematics teachers have neither knowledge of nor interest in either history or applications. They are merely docile folks who have *"learned"* algebra, geometry, trigonometry; and now "teach" the meaningless drivel—meaningless to themselves and meaningless to their pupils. Mathematics is frequently taught by football coaches, school principals, and science teachers, "because it requires so little preparation and paper correction on the part of the teacher." Indeed with a good "key" it is scarcely necessary to give a moment's thought to one's mathematics class except during the class period.

Unlike science and football and school administration, however, demonstrative geometry and quadratic equations are the same now as they were in 1895. Hence, there is little challenge to teachers or pupils to do other than to master the "important" and meaningless "truths."

Advocates of mathematics should face the questions of what mathematics and how much mathematics and what degree of perfection in mathematics are desirable, feasible, and justifiable for all thirteen-year-old children of .70 I.Q. and for every other age and grade and intelligence level of children. It is quite possible that if and when the advocates of mathematics face the realities of school education they may themselves recognize the futility of requiring every youth to "pass" courses in nonarithmetical mathematics. And if the mathematicians, with all of their prestige, ever do release their grip on the secondary school's core-curriculum and do advocate or even tolerate the application of common sense, school administrators may in turn relieve mathematics teachers from the impossible responsibilities for teaching mathematical principles and theories to youths who cannot or will not read or understand involved explanations of mathematical procedures.

If mathematical enthusiasts would ex-

amine Inglis's statements of four fallacies regarding direct subject values which are too often overlooked, more reasonable attitudes might replace their vested interests. Inglis distinguishes between (a) the fields of knowledge and skill that are of inestimable value to society and to civilization in their extended development though relatively few specialists, and (b) fields of knowledge or skills which are directly valuable for all or a majority of individuals; it is obvious that elementary arithmetic is all that could be included under (b). He distinguishes between the values of subject matter for production and accomplishment and values for consumption and appreciation; to balance a checkbook does not require that one be able to organize a bank. He distinguishes between certain and contingent values; much of our vocational and preprofessional applications of arithmetic and algebra are of remote contingent values. And he distinguishes between values attaching to certain parts only of any given subject, and the values attaching to the whole related field of knowledge as organized into a logical system; even though the ability to read graphs may be important it does not follow that one need solve simultaneous equations.[1]

If mathematics advocates would familiarize themselves with the findings of Rugg and Clark,[2] Wilson,[3] Wise,[4] and others, they might appreciate how little control of mathematics is required by a majority of people. Or if the mathematics enthusiasts would study the failure rates of mathematics throughout the junior- and senior-high-school period, they might be aghast at the death and destruction that prescribed and even recommended mathematics courses scatter in their trains. Or if the most outstanding zealot of the mathematics ranks would examine Chase's[5] findings regarding the paucity of mathematical knowledge and skill that is retained even by successful pupils after they have discontinued the subjects he must desist from his demands for prescriptions of algebra or geometry or business applications of arithmetic. For surely the slaughter of innocents is not justified in the vain effort to teach youths knowledges and skills for which they have no use and which even the few who succeed in passing the subjects do not retain.

We must not, however, assume that the advocates of mathematics as a required high-school subject lack energy or alertness or progressiveness—*of a kind.* Quite the contrary. It has, indeed, been almost unfortunate that some of the most progressive curricular modifications undertaken at the junior-high-school level should have been carried on by educators who have vested interests in compulsory mathematics and who have been, therefore, perhaps unconsciously, inhibited from examining their fundamental assumption that mathematics is and ought to be prescribed for all junior-high-school pupils of whatever level of intelligence or major interests. Hence, these energetic leaders have "enriched" junior-high-school mathematics by attaching to it physics, engineering, social science, elementary business training, and shop applications. But might we not be content to leave quantitative thinking and functional mathematics to the sciences and arts in which they normally develop?

The actual quantitative thinking and functional mathematics involved in natural and social science and shop projects may seem very slight to the mathematics specialist. If so, it may be because he has become immersed in his subject organization. More

[1] Cf. Alexander Inglis, *Principles of Secondary Education,* (Houghton Mifflin Company, 1918), pp. 389-392.

[2] Cf. Harold O. Rugg and John R. Clark, *Scientific Method in the Reconstruction of the Ninth Grade Mathematics* (Chicago: The University of Chicago Press, Supplementary Educational Monographs, Vol. II, No. 1, 1918).

[3] Cf. G. M. Wilson, Survey of the Social and Business Usage of Arithmetic, Teachers College, Contributions to Education, No. 100, 1919.

[4] Cf. Carl T. Wise, "A Survey of Arithmetical Problems Arising in Various Occupations," *The Elementary School Journal,* October, 1919, pp. 118-136.

[5] Cf. Sara Chase, "Waste in Arithmetic," *Teachers College Record,* September, 1917, pp. 360-370.

frequently, however, it is because mathematics teachers have so seldom done any applied quantitative thinking themselves that they do not even recognize it as mathematics when it is pointed out. In factoring and theorems and formulae they are engrossed. They dwell in a world that has no counterpart to them in reality. And because their symbolism is utterly meaningless to them, they feel sure that there must be some mystic values to their bablings.

If, however, intelligent and alert teachers can be found who have a spirit of adventure, elective mathematics begun as early as the seventh or eighth grade would become a series of joyous pioneerings. For pupils would not then have lost the sparkle for intellectual play such as characterizes true mathematicians.

If pupils have pursued *voluntarily* such challenging mathematical ventures as may be found in many of the recent alluring general mathematics series, they will have developed such a drive for mathematics that even conventional high-school algebra and geometry will be unable to abort it. The introduction of such elective courses in the junior high school would probably mean that those of us who are peddling bogus wares will have to seek new jobs. But eventually that will happen anyway. It is thoroughly unjustifiable to destroy life in order that futile men and women whose absurd education has resulted in an escape from reality into meaningless mathematical tricks which they do not understand—about the counterpart in reality of which they are not even curious!—may keep their jobs.

It is said that when an oyster gets a pain which irritates it constantly, a beautiful pearl may be formed. Some subtle belief that a pearl may result is probably the most constant factor which ensures the permanency of mathematics in the core-curriculum.

Nevertheless, had the oyster won the great battle with "Probably Arboreal" of which Don Marquis tells, perhaps the crustaceans would have refused thereafter to undergo the agony necessary to develop pearls for men! At any rate, American youths refuse to secrete a protective covering for the irritating mathematical substance. They just escape by becoming as superficial as most of their mathematics teachers. And like these teachers, the youths give their real attention to clothes and football and politics and automobiles. If they do not learn enough of the silly patter they are failed—but they grow no pearls!

P. W. L. C.

THE UNIT IN MATHEMATICS

E. R. Breslich

Editor's Note: *Mr. Breslich has long been identified with the reform movement in the teaching of mathematics known as "general mathematics" or "correlated mathematics." He is the author of numerous textbooks and is a member of the faculty of the school of education of the University of Chicago.*

J. R. C.

The topic to be discussed in this article is not new. Teachers have always been interested in units of instruction, but the meaning of the term is undergoing changes. In reading the literature relating to the unit in mathematics one finds various meanings assigned to the term. To some writers it signifies a block of work to be done within a specified time, to others it means a chapter in the textbook, and still others think of it as a method of instruction rather than one of organization of instructional materials.

According to the first meaning, the unit

might be the lesson for a day. To study the unit means to learn for each day; *i.e.,* to master the particular exercises, propositions, and processes assigned for the day. The goal of instruction and learning is the ability to recite in a satisfactory way the assigned lesson. In geometry each theorem is studied and mastered by itself without reference to the others. In algebra each process is learned to the exclusion of all others. Twenty-five years ago this used to be the common conception of the unit of instruction.

In discussing the "unit in demonstrative geometry for the ninth year" in the Fifth Yearbook of the National Council of Teachers of Mathematics, Orleans speaks of it as a "series of exercises which are based upon certain postulated geometric facts and which lend themselves to logical demonstration," to "cover in time the equivalent of six to eight weeks," and to be fitted into the work of ninth-year algebra. He organizes the unit under the following topics: preliminary definitions, vertical angles are equal, the congruence theorems, parallel lines, and similar triangles.

In the same yearbook, Seidlin uses the term unit to mean "the demonstration of a limited number of propositions . . . the principal purpose being to show the pupil what *demonstration* means." For the content he selects "a sequence of propositions expressly so chosen as to make the introduction to logical proof most real, most palatable, and least painful to the average pupil." Thus, logical sequence is the basis of organization in the unit as conceived by this writer. Both writers regard the unit as a collection of geometric principles suitable for instruction in the ninth grade and to be taught within a specified time limit.

Sumner in his *Supervised Study in Mathematics and Science* designates Euclid's books and the chapters of mathematical textbooks as the units of mathematics. Evidently he considers the unit to be a collection of mathematical facts, principles, and exercises in which logic is the basis of organization.

In view of the differences of opinion and the fact that the curriculum is constantly undergoing changes, the writer wishes to explain his own conception of the meaning of the term unit as it is used in more recent discussions.

In Euclid's time, the students of geometry were more interested in the logical demonstration of geometric principles than in any other phase of the subject. Hence, from the point of view of teaching and learning, it was natural that instructional materials were being organized logically and that Euclid made logic the basis of organization of his units which are known to us as the "books" of Euclid. He did this exceedingly well and his units have stood the test of 2,000 years. However, in view of the changes in the modern curriculum it is reasonable to ask whether Euclid's method of organization cannot be improved.

When logic is the sole basis of organization of a course, the pupil's daily lessons vary in difficulty, changing constantly and often abruptly from the most simple to the most difficult. If they are related to each other in any other way than logically that fact need not be made clear to the learner. Each new proposition is taught by itself and is followed by practice and a recitation as a test of understanding. The pupil finds it difficult to retain so many unrelated facts. As he realizes his inability his confusion grows. He does not know "what it is all about" and becomes a memorizer of lessons. The broad and really important principles of the course are lost sight of. The modern tendency in teaching geometry is away from learning merely the finished proofs of others in the direction of developing power to solve original exercises. It is for this reason that in the last thirty years

the leaders in mathematics have insisted that broad unifying principles should be made the core of the mathematical courses. The most frequently mentioned unifying factor in arithmetic, algebra, geometry, and indeed in all mathematical subjects is the function concept. Traditionally, our courses in algebra are organized in chapters which teach the pupil to add, to subtract, to multiply, to divide, to manipulate signed numbers, to solve equations, to solve problems, and to make graphs. These chapters have been the units of algebra. Each is taught and studied by itself. Each may go far beyond the present or remote needs of the pupil. He does not see the relationships between chapters. Nor does he see how each chapter contributes towards the course as a whole. His only aim is to finish the chapter and to have it over with. As he goes on with the course he develops the attitude of getting through. When he solves equations he does not want to be bothered with factoring and when he works with fractions he does not want to be interrupted with equations. Each chapter is studied intensively to the exclusion of all others. While he studies addition he adds anything from the simplest problem to one so difficult that it is found nowhere in the field of mathematics. He does not know what all this complicated work in adding leads to, and he does not care to know.

If we select unifying principles as the bases of organization of our courses, this may all be changed. Suppose we choose as one of the objectives of the course in first-year algebra the complete understanding of the function $y=ax+b$. To attain this understanding many experiences are required. For example one must be able to find the value of $ax+b$ for given values of a, x, and b. The pupil must therefore learn to make substitutions. He must acquire a knowledge of the laws of multiplication, addition, and subtraction for positive and negative numbers. He may tabulate the results and observe the changes of the value of $ax+b$ corresponding to the changing value of x. A graph of $ax+b$ makes the changes even clearer, for the graph readily shows the value of $ax+b$ for any particular value of x. It gives a concrete meaning to the coefficients a and b. It also answers the question of the values of x for given values of $ax+b$ including the value zero. This leads directly to the study of linear equations. Since $x=\frac{y-b}{a}$ some knowledge of fractions must be attained. Finally, it is possible to make up problems leading to $y=ax+b$, and some work in problem solving may be offered.

It is evident that to attain complete understanding of the function $y=ax+b$ the pupil must learn the operations with positive and negative whole numbers and simple fractions. He must know graphical representation. He must be able to solve simple equations and problems leading to such equations.

The illustration calls attention to one difference between a chapter organized logically and a body of materials organized as a unit. In the unitary organization the pupil sees clearly the relationship between the various divisions of the course and knows what each contributes to the course as a whole. No unit can be omitted without doing violence to the course. If one adds to the study of the linear function $y=ax+b$ that of the quadratic function $y=ax^2+bx+c$, it would be possible to construct a course equivalent to the traditional algebra, but much simpler to assimilate.

The foregoing example shows that a chapter is not necessarily a unit but that it may be changed to a unit by reorganizing the course on the basis of broad principles to which each chapter must make a definite contribution. A second illustration will now be given to show how the contents of a

chapter should be organized to transform it into a unit.

In his college mathematics, the writer had the good fortune of working under an instructor whose first assignment on a new chapter was always the whole chapter. To the students this seemed to be an unreasonable assignment but it turned out to be of great value to all of them. Having examined the chapter as a whole they knew from the first day on to the end of the chapter what it was all about. They saw how the work of each day contributed to the large unifying principles of the chapter. They recognized the mutual relationships of the instructional materials. They studied the chapter much more intelligently than would have been the case if each day's work had been assigned and learned by itself. As a result, all enjoyed the course and knew a great deal about it when it was finished. For example, the book contained a chapter on the solution of equations. It gave many theorems relating to the number of roots of an equation, rules of signs, graphical work, and ways of transforming equations, but each theorem was presented to be learned by itself without reference to the others. The recitation following the assignment of the whole chapter brought out many questions and answers. What does the chapter aim to teach? To enable the student to find the roots of rational integral equations of nth degree. What might be the nature of the roots of such equations? They may be real or complex; if real they may be rational or irrational; if rational, they may be integral or fractional. Some of the principles will enable one to find the integral roots, others to find fractional roots, etc. As the discussion went on, the students were led to see that each theorem had a purpose and therefore needed to be thoroughly understood. They saw at the beginning what usually remained hazy to them until the study of a chapter was finished.

The instructor might have gone a little further. He could have given a clear view of the chapter which all the students would have understood. This preview might have been as follows: "The chapter teaches how to solve equations like $3x^4 - x^3 + 4x^2 + x - 8 = 0$ which we shall call rational integral equations. The roots of such equations are either real or complex. If real, they may be rational or irrational. If rational, they may be integral or fractional.

"We shall first learn how to find the integral roots of an equation, and then take up the others in the order indicated. Our first problem calls for the use of certain principles which we must understand. You will find them in your book on pages blank to blank. Your first assignment will be to acquaint yourselves with these principles and to learn to use them." The instructor may have illustrated his statements in this preview by means of concrete examples to show what was meant. In the study of the reorganized chapter each student would have gone about his work intelligently, understanding the connections between the various principles and seeing the relationships of the theorems to the chapter as a whole. The reorganization of the chapter would have transformed it into a unit.

The foregoing illustrations have brought out some of the following characteristics of a teaching unit in mathematics:

1. It is a body of closely related facts and principles so organized as to contribute to the understanding of an important aspect of the course.

2. It must be possible to present the unit as a whole, in a form so concise as to give the learner a clearer conception of it before he undertakes to study it.

3. The objectives must be so definitely stated that they are clear not only to the teacher but also to the pupil. The learning products must be known.

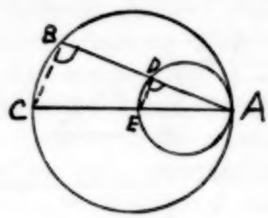
4. All pupils properly qualified to take the course must be able to master the minimum essentials necessary and sufficient to attain complete understanding of the unit. In addition to this minimum, the unit must contain supplementary material to allow freedom in adapting the work to the individual differences of the pupils.

5. It must be possible to devise tests which secure objective evidence of the understanding of the unit.

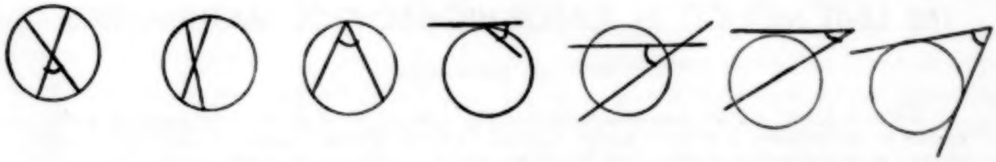
It must be clear from the foregoing discussion that the question of time and size does not enter in the organization of a unit. However, it is difficult for most high-school pupils to form a clear conception of a body of materials which it would take more than a month to assimilate. Experience seems to show that a unit which can be studied in three to four weeks is most suitable as to size. When it is not possible to finish it within that time the teacher should find a way of simplifying the unit either by transferring some of the materials to other units or by dividing it into two smaller units. For example, the materials relating to the study of the circle in plane geometry form too large a unit, but they may be reclassified to form smaller units like the following: the relationships between chords, arcs, and central angles; measurement of angles by circle arcs; relationships between segments of intersecting chords, secants, and tangents; and regular polygons inscribed in and circumscribed about the circle.

The advantages of unitary organization over the chapter or book organization may be illustrated with one of the units relating to the circle, the measurement of angles by means of circle arcs. The objective of the unit is to enable one to use the method of measuring angles by means of arcs in solving original exercises and in practical problems.

For example, in the figure below the angles at B and D are right angles because each is measured by one half of a semicircle. Hence, this fact may be used to prove BC || DE. The broad unifying principle of this unit is that "an angle whose



sides intersect a circle is measured by one half the sum of the intercepted arcs." The arcs may be positive, zero, or negative, which gives the following cases to be considered:



By moving the intersecting lines to different positions to make the point of intersection fall within, on, or without the circle, various cases are obtained which are usually learned as separate theorems, but which are now shown to be related to each other.

Furthermore, the proofs for the various theorems can be made practically alike. Each proof is therefore a test and review of the preceding. Thus, not only the theorems but also the proofs become easily the permanent possession of the learner. To be sure, not every unit can be organized so as to show the relationships as clearly as in the one given above, but in every case it is possible to improve upon the traditional organization in chapters or books.

Thus, in units on congruence and similarity, the objectives are not only the mastery of the separate theorems on congruent and similar figures, but to develop the power of using the principles of congruence and similarity in solving original exercises. In the unit on proportionality of line segments, the unifying principle is illustrated by a diagram in which any number of parallel lines are cut by two transversals.

The special cases are the following:

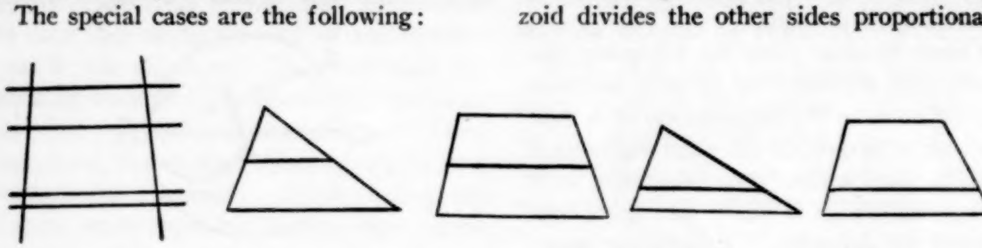
1. The segments on one transversal are equal.

2. One side of a triangle is bisected by a line parallel to a second side.

3. One side of a trapezoid is bisected by a line parallel to the base.

4. A line parallel to one side of a triangle divides the other two sides proportionally.

5. A line parallel to the base of a trapezoid divides the other sides proportionally.



It is possible to organize other units on parallelograms, areas, circles, and regular polygons in a similar way. The pupil to whom geometry is presented according to the unit plan will not fail to make the broad principles of the course his permanent possession.

THE UNIT METHOD IN JUNIOR-HIGH-SCHOOL MATHEMATICS

Butler Laughlin

Editor's Note: *Butler Laughlin went recently from the head of the department of mathematics to the presidency of the Chicago Normal College. He has conducted numerous studies of the Chicago Elementary Principals Club on experimental work in mathematics.*

J. R. C.

Junior-high-school mathematics requires a new and different approach from that used in the elementary school. The aims and viewpoint must be worked out to articulate with previous training, and yet meet the present needs of the child. The student has had six years of work in dealing with number combinations with a minimum of social application; to convince him of their value, we must show him their social utility. We cannot achieve this aim unless the units in the junior high school are organized for fundamental understanding with a minimum of drill work. Junior-high-school children care less for pure practice than in the earlier stages of mathematics, but they are much interested in units of work which bear a close relation to actual life.

These units, therefore, should always be organized around interesting social situations so that they will be broad, comprehensive sections of quantitative understanding with a minimum of drill. The organization of such a course must recognize that all the fundamental operations have not been mastered; nevertheless, drill units must be brought in with considerable care and only after the need for them has been well established. The junior-high-school youth will do drill work capably only as he feels that it is necessary as a part of a valuable activity.

The units must further be carefully selected and organized so as to be attractive to the adolescent child. If they are based on some of the many widely used business situations there is more likelihood of creating an active interest. There comes a time in the life of every growing child when he begins to be interested in some of the activities of adult life. Some adult activities do not appeal to children of junior-high-school age but there are many situations in the quantitative world which are interesting and at the same time valuable. These we must discover and use. It is impossible to cover all the situations which present themselves for discussion. For this reason the teacher should select not more than four units for each semester. A period of five weeks gives a class sufficient time to consider a valuable unit. Time so set aside to solve a few valuable social situations is time well spent. If the selection of these units is carefully done a student will be able to think through a few situations by the end of the semester, but not a dozen or so.

Great care should be exercised when the tentative units are set up, and all should be carefully tried out to see if they are teachable. A teacher often finds that a supposed unit is only a small topic. The unit must be broad and comprehensive but at the same time it must be interesting to the children. There are really two types of units to be considered in mathematics: (a) units of understanding and (b) drill units. The product in the first case is an understanding of the situation. This is unitary learning in that it is mastered or not mastered. But in the second case the product is variable. Skill may be good today but in a month from now may be almost entirely gone. If one masters a unit of understanding it is there to stay, but if he gets it by a drill process it may not stay with him.

Two divergent types of teaching are necessary for these two units. The drill work should be separated from the understanding and dealt with as a drill unit, whereas when we teach the unit of understanding we must remember that the student learns from the material and does not necessarily learn the material. In some of the drill units the actual material is learned as the combinations in addition or subtraction.

After the units have been selected the teacher is ready for the next step, which is to establish the learning situation. It is necessary that the unit of understanding be identified with life and that it be established in the minds of the class as having a place in our daily routine. It is not enough to be able to force a situation and get all to take part in the activity. There must be real hearty coöperation. One way to get started is to have a round-table discussion at which time the pupils are given a chance to tell all their experiences relating to such a unit. This may be done through leading questions by the teacher or by giving the children a chance for free talk. In any case the class should be given the opportunity to talk itself out before any work is started. When this technique is used each pupil is given a chance to start where he happened to be when the discussion ended. By the old method the more experienced pupil often uses only his past experiences. In the plan suggested he has used all these before the real work starts and he is generally stimulated to contribute something new.

Suppose the unit for discussion aims to develop the ability to read and construct graphs. The pupils should first tell of their experiences with graphs, so that in a period of forty minutes the work may be motivated. Suppose a unit on investments is up for consideration in which the purpose is to develop an understanding of stocks and bonds, and to develop the ability to solve some of the quantitative situations involved.

There should be a lengthy discussion to bring out the interesting experiences of the class and probably the related experiences of the parents. When such a discussion has been finished the lesson should be well motivated and the teacher oriented, ready for the next step.

The presentation follows the exploration. Here the teacher presents her contribution to the orientation. The best procedure is a well-organized lecture in which the teacher sets forth the unit in its simplest form. In the first unit mentioned above the teacher might well take several kinds of simple graphs and show the pupils just how they are read. This should be done step by step so there will be ample opportunity for the students to follow the procedure. This will result in an understanding but not a skill. The class must not learn the individual graph but should learn from the experience. When the presentation has been finished it should be followed by a presentation test in which the class writes out what the instructor has given out. The presentation should be repeated until the class understands it thoroughly. If the second unit, stocks and bonds, is used, there will probably be a series of problems solved by the teacher. This part of the work must be well done before supervised study, the next step, is attempted.

Supervised study is the most significant step because it is in this period that the real learning takes place. Here the child gets his experiences and it is here he must learn early that the experiences are given to learn from and not to learn. The assimilative material is to provide experiences and is not itself to be learned. During this period the room is a workshop with the teacher acting as supervisor. He passes among his workers helping where help is needed. If he finds a difficulty which is common to several people he will call them together for a conference on this point. He teaches where teaching is needed but will not do the work for the pupils. In order to find weak spots in the pupils thinking and performance, a few of the class papers should be collected each day and analyzed for mistakes. A few minutes at the beginning of the work period may then be used for clearing up difficulties. This will not, however, be in the form of a lecture, but will be for those who need help or for those who think they need some instruction. The class as a whole will be solving its own problems. Tests must be given at intervals during the supervised-study period to determine the progress of the class and to locate weak spots where more teaching is necessary.

A large part of the teacher's time should be given to selecting and organizing material of study. It is the lack of well-organized assimilative material which makes it difficult to organize work on the unit basis. Single textbooks do not contain enough material for any one unit, so that several texts must be on hand to furnish adequate material. Some units cannot be supplied from textbooks; in this case mimeographed materials must be furnished

When the supervised period has been finished, a mastery test of the unit should be given for both comprehension and understanding. This test must keep in mind a fundamental principle of the unit method —that understandings are developed because they are necessary in life activities. The examination must always show that the unit of work represents a section of the child's life. The understandings and skills which are developed must form a part of the necessary equipment of a child's life.

REORGANIZATION OF THE TENTH-, ELEVENTH-, AND TWELFTH-YEAR COURSES IN MATHEMATICS

GORDON R. MIRICK

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J. R. C.

The point of view governing instruction in a subject recognizes the importance of both the growth in the subject and the ability to use the subject either in other subjects or in everyday life. If a balance between these two aims is not maintained, instruction in the subject becomes either academic, narrow, isolated, or it becomes so general, so diffuse, that the logical and scientific is compromised. Stated briefly, the objectives in the teaching of any course in mathematics are:

1. To teach the subject matter that is sound from the standpoint of a mathematician
2. To teach that subject matter that is useful for a citizen

As it now stands our tenth-, eleventh-, and twelfth-year courses consist of year and half-year courses in plane geometry, solid geometry, intermediate algebra, advanced algebra, and trigonometry. In most schools, pupils are allowed to take solid geometry, trigonometry, and advanced algebra in any order. The work for the most part is academic and, further, the pupils fail to see the development of mathematics in the broader sense. For this reason alone, some consideration is now being given the problems peculiar to these years and to the work in the first-year college classes.

Any one attacking this problem of reorganization of the mathematics courses in the tenth, eleventh, and twelfth years must be conversant with the whole range of mathematics instruction, including in this range grades seven, eight, and nine; the work in the first two years of college and the needs of the persons engaged in various occupations such as engineering and business.

Among the numerous problems confronting the investigator in these fields are:

First, the consideration of who should be allowed to elect work in mathematics in the senior high school. Obviously, the pupils in the senior high school vary greatly in ability and interest in mathematics. Besides the pupils who have no trouble in mathematics in these upper grades, there are two groups of pupils who do not profit to the fullest extent. These two groups are:

1. Those who are below average in ability who profit little by the work
2. Those who are of average ability or above but who are not challenged to do their best work

If more consideration were given by teachers to the needs and interests of the pupils planning to elect courses in mathematics, the two groups mentioned above would be much smaller.

Second, the consideration of the materials of instruction for a year's course in plane and solid geometry.

Third, the consideration of the materials of instruction for a year's course in mathematics following a course in plane geometry or a course in plane and solid geometry.

Fourth, the consideration of the materials of instruction for a year's course in mathematics following the two last named courses.

The final solution of the third and fourth problems is dependent on the answer to the first problem and whether it is possible to

give a year's course in plane and solid geometry.

These courses of study for the eleventh and twelfth grades were undertaken by the writer in 1924. Pupils entering the eleventh grade had completed the junior-high-school course in mathematics and a year in plane geometry. Some tests showed that they needed a complete review of the algebra that they had studied in the ninth grade, but it was the feeling of experts in mathematics that this review could not be covered as well by a direct repetition of the work of the ninth grade as by a course organized on an entirely different basis. The points of emphasis in the reorganized course were:

1. The function concept
2. The development of meanings rather than the extension of manipulative skills
3. Contact with lifelike situations

It was found that after algebra had been covered through such topics as the fundamental operations, simple and fractional equations, and linear functions, the work on logarithms and trigonometry should be undertaken. Both of these subjects afforded a wide range of applications. In the work on quadratic functions, considerable emphasis was given to the extension of the locus concept developed in plane geometry. After the topics of progressions and the binomial theorem had been completed, a unit in economics and investments was taught, thus completing the year's work.

In the development of this course no consideration was given to the College Entrance Examination requirements but in the twelfth grade the majority of the class were scheduled to take the examinations for admission to college. The examination that pupils took was the Comprehensive Mathematics, CpH. This examination was divided into three parts—plane trigonometry, solid geometry, and advanced algebra. The pupils were required to take two or three parts. As it was desirable to continue with the study of the function concept, the work was planned so that the students took trigonometry and advanced algebra. Because of the necessity of meeting these examinations, the first part of the year was given over to calculus and the latter part of the year to the development of advanced algebra and the completion of trigonometry. Although most of the work in the calculus was taken in the first semester, a number of applications of it were made in connection with the work on theory of equations.

Following are the detailed outlines of the two courses.

ELEVENTH-GRADE COURSE OF STUDY

I. Functions
   1. Informal discussion
   2. Representation
      a) Line graph
      b) Bar graph
      c) Pictograph
   3. Simple formulas and their graphs

II. Classification of operations
   1. Fundamental laws of arithmetic and algebra
      a) Commutative
      b) Associative
      c) Distributive
   2. Addition, subtraction, multiplication, and division of simple algebraic expressions
      a) $a^m \cdot a^n = a^{m+n}$
      b) $a^m \div a^n = a^{m-n}$, $m > n$
   3. Simple equations and problems

III. Operations continued
   1. Fractional equations and problems
   2. Factoring as the need arises
   3. Fractions—formulas

IV. Linear functions and simultaneous linear equations
   1. Graph of linear functions. Solution of simultaneous equations by graph

2. Graph of $y = \tan \Theta$ where Θ varies from $0°$ to $360°$
3. The derivation of the type equations of a straight line
   a) $y - y = m\ (x - x)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$
   b) $y = mx + b$
4. Algebraic solution of simultaneous linear equations
5. Problems

V. Transcendental functions
1. Exponents and exponential functions
2. Logarithms
3. Trigonometric functions of the general angle
   a) Solution of right triangles by natural functions and by logarithms
   b) The Law of Sines
   c) Application of trigonometry to surveying and mechanics

VI. Quadratic functions
1. Graphical solutions
2. Algebraic solutions — radicals, imaginary numbers
3. Elementary properties of the following loci:
   a) Parabola
   b) Circle
   c) Ellipse
   d) Hyperbola
4. Extension of the locus work of plane geometry by means of this work in analytics

VII. Progressions—problems

VIII. Binomial theorem—problems

IX. Economics and investment

TWELFTH-GRADE COURSE OF STUDY

I. Review of progressions

II. Mechanics—the consideration of the notion of velocity and acceleration

III. Functions and their graphs—variation

IV. Differentiation of algebraic expressions
1. Fundamental formulas
2. Maxima and minima
3. Applications to mechanics
4. Small errors—measurement

V. Integration of simple algebraic expressions
1. Elementary notion—indefinite integral
2. Applications
   a) Areas and volumes
   b) Mechanics—force, work, etc.

VI. Imaginary and complex quantities:
1. Rectilinear and polar representation
2. De Moivre's theorem

VII. Theory of equations (some use is made of the calculus)

VIII. Determinants

IX. Trigonometry completed

X. Scales of notation

XI. Permutations combinations
1. Probability
2. Statistics

TENTH-GRADE COURSE OF STUDY

After it had been decided that classes in the tenth grade should be taught a course in plane and solid geometry instead of the usual course in plane geometry, considerations were given to the method of approach. One method was to teach plane geometry for two thirds of the year and solid geometry the last third of the year. This method of approach was not used because it fails to show and to take advantage of the close relationship between the two phases of geometry. First the minimum essentials for a course in plane geometry and the minimum essentials for a course in solid geometry were decided upon. In the preparation of these lists of minimum essentials, the objectives in geometry teaching were

constantly kept in mind. Then the two lists of essentials were combined into seven teaching units, the first two units containing only plane geometry. The units are as follows:

I. Introduction to plane geometry

II. The idea of proof, congruence, and parallel lines

III. Quadrilaterals—locus in a plane—introduction to solid geometry—lines perpendicular to a plane or to a line; lines parallel to a plane, parallel planes

IV. Circle, angle measurement, elementary properties of a sphere, dihedral angles, perpendicular planes, locus in space

V. Area, theorem of Pythagoras, lateral areas of prisms and pyramids, volumes of prisms and pyramids

VI. Similarity, applications to solid geometry

VII. Circumference and area of a circle; lateral areas of cylinder, cone, and sphere; volumes of cylinder, cone, and sphere.

In giving these outlines, the writer hopes that the reader will realize that the courses are not fully developed. Each year sees many changes.

IMPORTANT CONTRIBUTIONS OF THE NEW MATHEMATICS WITH SPECIAL REFERENCE TO THE JUNIOR-HIGH GRADES

J. Andrew Drushel

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J. R. C.

Out of the development of standardized computing and reasoning tests and in connection with the rise of the new educational psychology came much experimental teaching and research, resulting in a different type of mathematics as regards content for the pupil and in technique for the teacher. The new mathematics is the mathematics designed for immediate use which should be projected into modified and not too remote future values.

To have value mathematics must be pitched on the intellectual level of the student rather than on the mechanical memory level and the student must *want to* master the particular mathematical job he undertakes. He may want to master mathematics because he sees an immediate or a near future need for it or because he is interested through previous experience to know what it is all about.

Good teachers create interests for their pupils whether these be adults or children. If we should trace the origin of our own present interests, more often we would find their source outside of us than inside of us. Content in mathematics should not be selected solely on the basis of children's needs and their interests but also on the basis of interests which teachers can create for pupils.

It is here suggested that the junior high school does not have a monopoly on the new type of mathematics. Several new arithmetic texts furnish abundant evidence that the new type begins with the first grade, continues through the elementary and junior high school, and is slowly finding its way into the senior high school.

Contribution One. Standardized diagnostic, timed practice, and inventory achievement tests in computing and in reasoning.

As an outcome of the use of these tests modern method insists: (1) that we receive pupils entering junior high school where they actually are in their mathematical experience, not where the course of study says they ought to be, nor where we who are to be their teachers think they ought to be; (2) that we take them along the mathematical road as rapidly as they can travel happily and profitably.

This procedure is contrary to the traditional practice of kenning those who can and "canning" those who can't.

Where the pupils are may be determined by diagnostic tests; how fast they can travel at the various levels is determined by timed practice tests; whether they have reached a set goal is determined by inventory tests.

In relation to arithmetic where are the members of the lowest quartile of those entering junior high school?

In the fundamental processes with integers they are from one year to two years below the Courtis standards for the end of the 6th grade. They have little or no understanding of the processes involving either like or unlike fractions. As a group they will fail in the addition of two fractions such as $\frac{2}{3}+\frac{3}{4}$ 2 times in 3 attempts; they will fail in the subtraction of such examples as $\frac{3}{4}-\frac{2}{3}$ in 50 per cent of their efforts; in multiplication of such examples as $\frac{2}{3}$ of $\frac{3}{4}$ they will fail in one out of 3 attempts; and in division examples such as $\frac{2}{3}\div\frac{3}{4}$ there are 2 failures in 5 efforts. They have learned as yet very little in problem solving beyond the simple one-step problems of the 4th grade.

It must be kept in mind that we are dealing with a group whose individuals are retarded one or two years or more in arithmetic when they enter junior high school and that this retardation relatively speaking increases as they pass through the junior high if proper provision is made for the upper group.

Contribution Two. Different content for different groups of the same school grade.

In the case of the lowest group, arithmetic must be made the core of the mathematics curriculum for grades 7, 8, and 9. Much of the content can be drawn from regions that are new and interesting to the pupils, but in respect to difficulty such matter must be pitched on the level of 5th-grade pupils of normal ability and possibly lower. With this content should be used new instruments of instruction and of manipulation such as the graph and the equation. The second book of the 3-book edition of several of our modern arithmetic texts is a more suitable text from the standpoint of content for the lowest third of our present junior-high-school population than any one of the dozen junior-high mathematics now on the market and I do not hesitate to include our own series in each of these categories.

It may be remarked that for the middle and upper groups the new type of mathematics makes provision:

1. for the extension of arithmetic into the 9th grade by maintaining through suitable practice the computing skills already acquired and by applying arithmetic to much so-called algebra material to secure an appreciation of the relative merits of algebra and arithmetic

2. for bringing much new material from the fields of algebra, geometry, and numerical trigonometry into the 7th, 8th, and 9th grades, much of which can be attacked on an arithmetic basis

Contribution Three. Problem solving.

This contribution relates to problem solving and may be broken into two parts for discussion: (1) problem attack; (2) manipulatory instruments.

The most difficult part of elementary mathematics from the standpoint of securing satisfactory results is problem solving. Practically all pupils have more or less dif-

ficulty in this field of endeavor. The chief causes of this difficulty may be traced to three sources:

1. Pupils too frequently rely on formal rules through sheer memory processes. This is a heritage from our traditional method of teaching problem solving.

2. Pupils have not learned to read problems. Possibly many of the lowest quartile do not have the native ability to do the analytical reading required in a two- or three-step problem put into verbal form.

3. Pupils have not learned a proper method of attack after the problem is read. It is at this point that the new mathematics is making a contribution very much worth while to all pupils learning to solve problems of the two-step or three-step variety by developing a technique which lays more emphasis on the function concept than has been done heretofore.

Ligda in his *Teaching of Elementary Algebra* recognizes the importance of the function concept in problem solving in language somewhat as follows:

> We cannot perceive the relationship between quantities that are not familiar to us. Since we are not able to perceive them we cannot express them in mathematical language. This principle leads us inevitably to one conclusion: we cannot express in mathematical language something that we cannot first express in plain English. We cannot express verbally or symbolically something that we do not perceive and the final conclusion is we cannot look for relationships before we are told what relationships are and how to find them in verbal statements.

The pupil should be taught from the beginning that learning to solve problems is an important task. Skill in problem solving is a conquest, a conquest of sufficient importance to demand and necessitate the organization and planning of an attack if satisfactory results are to be obtained.

In the case of two or more step problems the pupil should be taught to find first the large main relationship, and then to attend to the minor ones because:

1. There is such a thing as the heart of a problem.

2. Educational psychology generally favors learning in wholes, rather than in parts.

3. In reading an involved sentence analytically, the main clause is sought first and then the subordinate ones.

4. In mathematics symbolical expressions such as $3\times4+8\div2$ are interpreted first as wholes.

This means that the main relationship in an arithmetic problem when it is discovered can be expressed in terms of one of the four fundamental processes. In other words the problem is primarily one in addition, or one in subtraction, or one in multiplication, or one in division. By this procedure there is developed ability to translate problems into their mathematically fundamental relationships.

When the primary reasoning on a problem is accomplished then the solver must select an instrument through which to express and reduce to simplest form the relationship discovered.

The junior-high-school pupil who is trained in modern mathematics has a choice of one of three instruments (equation, graph, table) or he may use all three for purposes of developing skill in making his choice. Our lowest group should be trained to use these three instruments, but in much easier content than would be given to the middle and upper groups.

Of these manipulatory instruments the value of the equation in the writer's opinion cannot be over stated. In Thorndike's *New Methods in Arithmetic* the following paragraph is found in connection with his presentation of the value of the equation in primary arithmetic.

"The equation form is the simplest and clearest way to state a quantitative problem. It is one of the best ways to retain arithmetical facts in memory. Its use stimulates and trains the habit of inspecting obtained results to see if they do really meet the stated requirements. It prepares the pupil to understand formulas of all sorts. It is a model for brief, clear, and decisive thinking."

In connection with graphs it is probably proper to state that the lowest group will profit more by a wide and thorough acquaintance with statistical graphs than by covering the entire field of graphs as usually presented in a junior-high mathematics course. An intensive treatment of the algebraic (or so-called function) graph should be given only to those who have elementary-algebra ability.

Contribution Four. New types of content.

Modern mathematics is supplying much matter that should be treated in a laboratory manner.

1. Pupils should make and interpret correctly many measurements, such as the difference between 4 feet and 4.0 feet.

2. Pupils should learn how to use measured numbers in computation. To do this they need to learn that a computed result cannot be more accurate than the least accurate of the data used.

In the interest of accuracy it is just as necessary for a pupil to know the meaning of measured number as it is to know the meaning of counted number. In order for the pupil to know how accurate his result is, aside from computing accuracy, it is necessary for him to know how to apply the fundamental processes to measured or approximate numbers. It seems that this is just as fundamental content for all seventh graders as the addition and subtraction facts are fundamental for second graders.

When we grasp the significance of measured numbers more fully, there are three types of problems whose schoolroom solutions will come to correspond more closely with life conditions than they do now in many places.

1. Addition of decimals: $4.8+3.26+8$.

An impossible example unless the numbers are rounded off to the least accurate one so as to read $5+3+8$.

2. Find the area of a rectangular card 18.5 inches by 12.5 inches.

This means that the card is measured to the nearest tenth of an inch, that is to three-figure accuracy. Therefore the area can be found to three-figure accuracy. That is the nearest approximation that can be made. The area is 231 square inches, computed as accurately as can be done from the given data.

3. Find the circumference of a circle whose diameter is 25 feet.

This means that the measurement is made to the nearest foot and therefore the answer can be no more accurate than the measurement, that is to two-figure accuracy.

The circumference$=25\ \pi$ ft.$=78$ ft.

In this case the value of π may be taken as 3.1.

In conclusion it may be pointed out that in problem solving in junior-high mathematics the solution of particular problems leads to the statement of a rule or to the solution of the general problem either of which should result in a formula. This formula constructed by the pupils under teacher guidance is then used by them in solving other related particular problems and later is transformed into other formulas which in turn are used in solving particular problems.

HOW SHALL THE OBJECTIVES OF MATHEMATICS FOR THE SECONDARY SCHOOLS BE DETERMINED?

RALEIGH SCHORLING

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J. R. C.

This brief discussion is concerned with three questions: (1) Why should objectives be determined? (2) What is the scientific method as applied to an educational problem? and (3) How is the method being applied to the problem before us?

NEED FOR OBJECTIVES

The public invests large sums in order to make changes in children with respect to mathematics. Studies of the specific changes being attempted show an astonishing lack of agreement. So great was the variation shown in Guiler's investigation[1] of thirty courses of study, that he secured his objectives in arithmetic for the elementary grades by accepting each item found in any three courses of study. In 1925 the writer reported an inventory[2] of the content of seven series of mathematics textbooks in the junior-high-school field. For a given grade he found wholly different offerings; e.g., units of arithmetic, algebra, statistics, intuitive and demonstrative geometry and trigonometry. Probably the situation has become more uniform in these grades in recent years, but at that time there was not even agreement concerning the large topics or fields to be covered. In one textbook for a particular grade there were presented, under two topics, discussions of 42 business forms—traveler's check, bill of lading, coupon note, and the like—and yet some of the books for the same grade taught not a single business form. There was also very little agreement with regard to the details—concerning, for example, what specific skills in percentage should be established to, let us say, a ninety per cent mastery.

The greatest variation in mathematics is the product of teachers with little or with inadequate special preparation. In California and Michigan, for instance, teachers with a secondary-school certificate may, and do teach arithmetic in the elementary schools with no preparation other than their own early work as pupils in these grades. Then, too, from 40 to 60 per cent of the beginning teachers in high school teach subjects which they have never studied in college. During the past six months the writer has visited the schools in a considerable number of college towns where one might expect to find the best samples of the teaching of arithmetic. To be sure, much excellent work was observed; but one reaches the conviction that courses of study are recklessly made, and are far too often carried out by teachers without special preparation and with little or no supervision.

As this sentence is being written two pupils are observed in a struggle with homework in arithmetic. The fourth-grade pupil is attempting to learn multiplication of whole numbers with the multiplicand and multiplier each having two digits. His thinking is hopelessly confused by the teacher's insistence that he check each problem by a method which was long ago deleted from our better textbooks and which, in fact, is far more difficult to learn than is the fundamental process of multiplication.

[1] W. S. Guiler, a doctor's dissertation on file at the University of Chicago.

[2] Raleigh Schorling, *A Tentative List of Objectives in the Teaching of Junior High School Mathematics* (Ann Arbor, Michigan: George Wahr, 1925), vi+140 pages.

The eighth-grade pupil is struggling with an assignment in which he is expected to discover tests for the divisibility of integers by 2, 3, 6, 8, and 9. There is not the slightest evidence that either the course of study or the teacher have been affected by the report of the National Committee on Mathematical Requirements. A parent cannot predict what will happen to his child in arithmetic. In contrast with other school subjects, arithmetic seems to make a very good showing, however, for 60 to 70 per cent of the items in courses of study are constant. But it is the 30 to 40 per cent of variable elements which cause wasted effort on the part of the teacher and confusion and low mastery on the part of the children. Teachers need and desire that objectives be specific and that they be determined by the best methods now known.

THE SCIENTIFIC METHOD

It is assumed that a scientific answer is desired if one can be obtained. There are several extreme reactions to the scientific method when applied to educational problems. There is, first of all, the confused teacher who has a vague notion that the scientific method involves some kind of fancy statistical manipulation. Then there is the youthful teacher who has perhaps studied education during a summer session or two and who seizes too gullibly and eagerly upon all that is turned out in the name of the science of education. Finally, there is the ultraconservative teacher with little recent training who is certain that no educational problems will ever yield to scientific treatment. This latter type of teacher has the mistaken notion that the subject matter in the so-called sciences of physics, chemistry, medicine, and the like, is the all important thing, and that the items of subject matter are fixed for all time—that is to say, are known with absolute certainty. The fact, of course, is that the items of subject matter in these sciences are undergoing extensive modification. Recently one of the leading chemists of the world remarked to the writer that in his judgment at least half of the material now in the chemistry textbooks used in our secondary schools is fiction. A man prominent in medical circles was asked this question: "What percentage of the practice of medicine rests on a scientific basis?" He replied, "One can only guess; but as a maximum, surely five per cent is a generous estimate." I have often suspected that doctors, when they diagnosed my ills, were guessing; but I did not expect them ever to admit it.

The most important characteristic of science is its method. The essence of the scientific method is the examination of all discoverable facts and sources to make sure that we are as nearly right as possible before we go ahead. Only in that way will we achieve a decreasing dependence on rules and traditional practice and an increasing habit of employing the techniques of the older sciences as we develop a method of solving educational problems that is unique to the field but nevertheless scientific in its ideal.

THE CRITERIA

What specific facts of mathematics shall be taught to the point of mastery to our pupils of the secondary schools? What specific skills and attitudes shall be fixed? What specific concepts shall be developed? The traditional classification of the aims of mathematics (practical, disciplinary, and cultural) is, although valid, so inadequate as to be meaningless as a guide to the teacher in the selection of material. The following eight criteria are proposed as being useful in the choice of the specific changes that are to be made in children.

1. *A Summary of Social Needs.* What mathematics do pupils need to know in order to succeed in the science courses in

the high school or in the early years of college? What elements are really important for the next course in mathematics? What items of mathematical knowledge are necessary for the well-informed reader who, considering the enormous growth in the population of the secondary school, now represents perhaps a tenth-grade level?

Beginning studies have been made, of course, in the efforts to answer these questions; but there has been no adequate summary of the unpublished studies now filed in the various libraries, and these studies have been neglected by most of the writers in the field. In fact, in the recent books written for the training of our mathematics teachers there is almost no trace of the many important and available investigations. It may be argued that such studies include much chaff, but this criticism does not justify the total neglect of the occasional grain. It is to be hoped that the National Council of Teachers of Mathematics will, at the earliest possible date, publish a yearbook that will summarize the important studies in order that the results may be used as a criterion in modifying the courses of study and the textbooks.

2. *Guiding Principles Supplied by a Philosophy of Education.* The road from practised intuition and common sense to science is a continuous one, and philosophy is one of the milestones on it. The philosopher is concerned with values. No item should be, and perhaps none can be, accepted for a curriculum until one has answered such questions as: Is it worth while? and, For whom? The philosopher sees that certain shots sent towards the educational target are wide or short or long. He investigates causes. He may even do a little tabulating. He begins to classify, to arrange, and to organize. He formulates an educational platform of philosophic principles. In spite of the great variation in practice in our secondary schools, certain philosophic principles are receiving wide acceptance and are serving as guides in the shaping of our curricular materials. An illustration of the application of this criterion is furnished in the "Report of the Sub-Committee on Junior High School Mathematics."[3]

3. *Guiding Principles Supplied by Educational Psychology.* Another milestone on the road from common sense to science is psychology. When the philosopher becomes interested in how people learn, he looks hopefully to psychology for assistance. As you well know, there has been great activity in the field of educational psychology in recent years. Out of the numerous investigations in the laboratories and classrooms certain principles have, for the moment at least, won general acceptance. No objective should be included in one's list if it is in conflict with a valid principle of educational psychology. The brilliant English schoolmaster, T. Percy Nunn, was among the earliest to apply educational psychology to the selection of mathematical materials; and in recent years mathematics in America has been particularly fortunate in that there have been honest efforts in recent textbooks to apply this criterion.

4. *The Response of the Learners.* Probably the best criterion, though little used, is the pragmatic one of extensive classroom trial. Given a tentative selection—a long list of detailed items—what ones can, in fact, be mastered by children? What ones seem to contribute to growth in mathematical power? To what extent does the pupil accept each item as something worth doing? And, finally, is the task one which he enjoys doing? The criterion amplified by these questions implies that the experimental material needs to be taught by teachers who are interested in learning, who keep systematic records, who are able to draw valid inferences, and who will make periodic reports to the investigator.

[3] *North Central Association Quarterly*, II (March 1928), pp. 9-32.

5. *An Inventory of Courses of Study.* It appears that teachers in general neglect courses of study. Recently the writer visited a school which prides itself on having an excellent course of study in arithmetic. Although the course was only a year old, seven of the ten teachers could not locate their copies of the course of study. It is the writer's opinion, furthermore, that in the whole country there are not as many as ten excellent courses of study in mathematics. Nevertheless, such model and progressive courses as one may find in Denver, Fresno, Chicago, and the State course of study for West Virginia may well serve as instruments to improve the mathematics teaching in our schools. The study by Guiler, to which we have already referred, should be repeated at an early date in order that teachers may have available as a guide a composite of the best practice.

6. *An Inventory of Recent Textbooks.* As has already been stated, the writer in 1925 reported an investigation of seven sets of textbooks in junior-high-school mathematics. This study should now be repeated, and a similar study should be made of textbooks in arithmetic and in each unit of work in the senior high school. These studies should be paralleled by inventories of the corresponding textbooks published ten or twenty years ago. Indeed, survey charts of this type are now in existence as confidential properties of several publishing companies, and it is to be regretted that these valuable documents showing the significant trends in the mathematics curriculum are not generally available to teachers.

7. *The Reports of Standardizing Agencies.* The reports of committees have been very influential in shaping the curriculum. The report of the National Committee on Mathematical Requirements was epoch-making in that it was the first report which represented extensive coöperation in a mobilization of thought with regard to a particular subject. The report was unique, also, in that it attempted to list the specific objectives for the junior high school in greater detail than had ever been done before. Nearly a decade ago the report of this committee was an excellent criterion for the selection of mathematical materials. In all probability the things most worth while in it will persist in our curricula. The converse is unfortunately not true. The report, though still accepted as a guide by many schools, appears to the writer to be in serious need of revision. Ultraconservative communities will find this report a valuable guide for still another decade, but progressive schools are eager to take the next steps in advance.

8. *The Jury System.* In determining objectives, the judgment of competent and experienced teachers is valuable; but it is of the utmost importance that these judges be highly selected, relatively few in number, and entirely willing to undertake the arduous task of passing judgment on each of the many specific items in the curriculum. The value of the outcome of the jury system probably decreases rapidly as the number of judges is increased. There is perhaps no more futile effort in the field of determining objectives than the wholesale collection of opinions of teachers who too frequently give back to instructors of brief courses in summer sessions the materials which these instructors have passed out to their students. If a dozen of the most gifted teachers of mathematics in America could be assembled for a continuous meeting of several months, the outcome would no doubt provide one of the most valuable criteria for the selection of objectives. One of the reasons why the report of the National Committee was so valuable some years ago is that it employed the jury technique. To some it will doubtless appear that the acceptance of this technique negates any claim of a scientific method. To the

writer, the judgments of competent teachers will always be an important factor in the scientific method as applied to educational problems.

CONCLUSION

It is the thesis of this discussion that the scientific method requires the use of every available source to make sure that we are as nearly right as possible before we go ahead. Finally, it is believed that if the specific items of information, skills, attitudes, vocabulary, and concepts which constitute the various courses in secondary-school mathematics were checked by the eight criteria here suggested, the outcome would be a list of objectives as valid as we at present know how to make them.

THE HISTORY OF MATHEMATICS IN RELATION TO ELEMENTARY CLASSES

Vera Sanford

Editor's Note: *Miss Sanford, professor of mathematics in Western Reserve University, has contributed numerous articles on the history of mathematics and on experimental work in the teaching of mathematics. The subject of her doctor's dissertation was "The History and Significance of Certain Standard Problems in Algebra." She is associate editor of the* Mathematics Teacher.

J. R. C.

The suggestion of using topics from the history of mathematics to enrich the materials of the high-school course is not without precedent in American schools today. The Report of the National Committee on Mathematical Requirements specifically recommends this practice in both junior and senior high schools. Writers on the teaching of mathematics urge this study, and authors of textbooks insert portraits of mathemeticians and historical notes in their algebras and geometries. But the selection of the material and the manner of its presentation are in general left to the teacher.

THE PURPOSE OF THE HISTORY OF MATHEMATICS

It should be understood at the outset that the object is to make mathematics a more interesting and significant body of thought rather than to give a survey of its history for the sake of the history itself. The student should come to the realization that mathematics is dynamic not static: it has changed in the past, it will continue to develop in the future. The concepts and symbols with which we deal are the result of a long period of human endeavor. Our difficulties in mastering them may perhaps be but the reflection of the difficulties of our predecessors in evolving them. Further, since in this work we, like Newton, may be said to be standing on the shoulders of the giants, we should know who those giants were and what they accomplished.

THE SELECTION OF MATERIAL

In choosing the topics to be considered, the teacher must keep the pupil's background clearly in mind. The minutiae which are important to the scholar are out of place in the high school. On the other hand, it is often essential to include details which, with an adult, one would either take for granted or else pass over without comment. In telling the story of Thales, for example, we should not puzzle the student with the discussion of whether certain theorems are specifically due to the " father of Greek geometry " or not although we should be clear as to the value of the method he is said to have used. But if we repeat the story of the cornering of the olive market, it would be well to indicate the importance of the olive crop in the days of Thales. Instead of being merely the embellishment of a dinner or furnishing an ingredient of

a salad, it should be remembered that at that time olive oil furnished the principal fat used in cooking. Neither butter nor soap was then known, but olive oil served for both. And, as if this were not enough, olive oil was used as the fuel for lamps also. Thales was a shrewd business man if he actually cornered this market by buying up the olive presses.

We should, on the other hand, guard against the perpetuating of absurdities. Fictitous tales of the origin of Hindu-Arabic numerals might be mentioned in this connection.[1] The late Professor Cajori characterizes these as "entertaining illustrations of a pseudoscientific imagination, uncontrolled by all the known facts."

Further, the questions children ask lead one to suppose that they are in general more interested in the "what" or the "how" rather than the "who" and the "when." In any case, we are seldom interested in a thing of whose existence we are unaware, and to be used successfully in the high school, work in the history of mathematics must be so arranged as to create a curiosity about the subject and a desire to study it further.

THE ARRANGEMENT OF THE WORK

In the attainment of the objectives given above, it is evident that a scheme adapted to the interests of the senior high school will not fit the needs of the seventh-grade student and vice versa. It is desirable then to consider the work on different levels: informal use of the history of mathematics as an integral part of the student's work in the seventh and eighth school years, and indeed in the intermediate grades as well; incidental use in the ninth grade and in the senior high school; systematic study in the college or teacher-training institution. This arrangement shows an orderly sequence from the time when the student scarcely realizes that he is studying the subject, through the conscious use of the material in discrete units to its chronological, racial, and topical study. The interests created in the first stimulate the rest. The insight gained in the second makes the third to be desired.

INFORMAL TREATMENT

Among the topics that have been used successfully in connection with the sixth grade are Egyptian numerals, fractions and computation in connection with the study of Egypt; the calendar, clocks, and sundials in connection with a time unit; the keeping of records by means of knots and tally sticks; and the use of the loose counter abacus in connection with the history of records. This material was not studied as mathematics but it appeared to the students simply as valuable information illuminating the rest of the work.

This informal use of the history of mathematics should be continued in the junior high school. Suppose for example we include in our study of weights and measures the probable origin of such units as the foot, the yard, and the mile; the standardization of these units due to economic necessity; the preservation of the standards in the safest places people had; the clauses relating to standard measures in the Great Charter (1215) and in the Constitution of the United States; the making of the metric system with its tale of adventure; and the work of the Bureau of Standards. In connection with such a unit, a seventh-grade class made drawings to illustrate the various units—the three barley corns from the middle of the ear whose length made an inch, King Henry I of England stretching out his arm to indicate the length of the new standard yard, a Roman soldier pacing out the *mille passuum* that was to become the mile, and Köbel's description of a rod as given in 1514:

To find the length of a rod in the right and

[1] Florian Cajori, "Fanciful Hypotheses on the Origin of the Numeral Forms," *The Mathematics Teacher*, XVIII, March 1925, pp. 129-133.

lawful way and, in accordance to scientific usage, you should do as follows: Stand at the door of a church on Sunday, and bid sixteen men to stop, tall ones and small ones as they happen to pass out when service is finished; then make them put their left foot one behind the other and the length thus obtained shall be the right and lawful rod to measure and survey land with and the sixteenth part of it shall be a right and lawful foot.

We consulted an arithmetic published in America more than a hundred years ago and found that in comparison, our task in learning to use tables of denominate numbers[2] is slight indeed. We investigated the question of the adoption of the metric system and the abandoning of English measures and discovered that the problem was far more involved than many of us had believed.

As an example of the use of the history of mathematics in an eighth grade, let us consider the work in indirect measurement and numerical trigonometry frequently allotted to that year. A class grew restless after lessons devoted to the textbook problems in finding the heights of trees or the widths of rivers by the use of triangles which they drew to scale, and they begged to know whether any one ever really measured heights and distances this way and, if so, what instruments they used? The question could be answered by books at hand which contained reproductions of pictures in sixteenth- and seventeenth-century works on surveying[3] showing grown men using just these methods with instruments that appeared to be very simple. It would have added to their interest had they known that, in at least one instance, the picture showed an open square in the city where the author lived, showing it so exactly that a tourist in Bologna today sees the same façades and the same fountain, although a modern building prevents the distant view of the two leaning towers, visible to Bettini from that place in 1641.[4]

The study of these pictures led to the reproducing of the instruments. Other pictures were found and new details were used. Dealers' catalogues were consulted to see whether instruments of the same sort were used today. The class formulated problems to be solved in the school building or in its vicinity. At the close of the work, we prepared an exhibit of our own handiwork; drawings of old-time instruments, photographs of our surveying parties at work, reports of surveys; and to ensure that the observer would realize that they were busy with mathematics of the usual type also—their textbook and their arithmetic drill book.

In the course of this work, we learned of Dr. della Corte's discovery of a surveyor's shop in Pompeii where were found the man's inkpots and rules and compasses for his permanent records, his wax tablet and stylus for field notes, and, most important of all, the metal parts of his *groma* or surveying instrument. We studied astrolabes, read or had read to us parts of Chaucer's account of this instrument written in English for the benefit of his ten-year-old son and carrying the title *Tractatus de Conclusionibus Astrolabii* (*Bred and Mylke for childeren*).[5] Astrolabes led us to the early transatlantic navigators and we wondered at their daring in sailing across the ocean with no more accurate means than this of telling where they were.

Taken altogether, the work involved industrial arts in the construction of the more complicated instruments. It touched English literature through the association with Chaucer. It led into the history of this country in the period of the explorers and discoverers. Examples of the instruments

[2]A list of these is given in the writer's *Short History of Mathematics* (New York: Houghton Mifflin Company, 1930), p. 360.

[3]*See*, for example, Miss Gugle's *Modern Junior Mathematics*, Wentworth-Smith-Brown, *Junior High School Mathematics*, and Smith, *History of Mathematics*, Vol. II. Pictures that have been especially helpful in this work appear in the writer's *Short History of Mathematics*.

[4]*Ibid.*, p. 245.

[5]*See* W. W. Skeat, *Complete Works of Geoffrey Chaucer*, III (Oxford: Clarendon Press, 1894).

in such pictures as Holbein's portrait of Nicolas Kratzer,[6] who was astronomer royal under Henry VIII of England; or of the bas relief of Ptolemy representing astronomy on the campanile in Florence,[7] brought us definitely in touch with the history of art. The accounts of our surveying trips provided an opportunity for precise and formal description of the projects. Meantime the work itself had carried us rapidly through the solution of the textbook problems, where similar triangles were used, to the study of trigonometric ratios. In fact, one of the problems chosen by the class necessitated the use and therefore the study of the sine ratio which was not included in the curriculum for that grade. In spite of the variety of topics included, the work was completed in the normal time for this textbook material in the school.

The objection will be raised that such work could as well be done with modern instruments as with old-time models of home manufacture with their inevitable crudities. It is true that the work was crude. But junior-high-school work with commercial instruments is crude also, for the student's technique fails to realize the finer possibilities of the transit or level. It must be understood that the modern instruments were studied in detail and much work was done with a transit constructed from three protractors and parts of an erector set.

With these illustrations of the way in which historical materials enrich sections of the work of the sixth, seventh, and eighth grades in mind, and with the assurance that these hindered the regular work of the course slightly, if at all, the value of the work should be considered from another angle. The diverse activities connected with the surveying unit, for example, challenged the interest of the mechanically-minded students who were here in a position to use their gifts to good advantage. Others contributed through bringing pictures from magazines and the Sunday papers ranging from the armillary spheres of advertisements, based on historical material, to Byrd's gyrocompass.

[6] See the author's *Short History of Mathematics*, p. 246. Notes on this astronomer and his work appear in R. W. T. Gunther, *Early Science in Oxford*, I, pp. 101-106. (London: University of Oxford Press, 1922).

[7] *Short History of Mathematics*, p. 293.

REFERENCE MATERIALS

During the past decade, the materials available in English for the study of the history of mathematics or for the use of high-school students have been enriched by the publication of Professor Smith's two volume *History of Mathematics*, and by the *Source Book in Mathematics* which he edited, the late Professor Cajori's *History of Mathematical Notations* in two volumes, Professor Karpinski's *History of Arithmetic*, a number of special studies as for example the Chace volumes on the *Rhind Mathematical Papyrus*, Gunther's monumental *Early Science at Oxford* (volumes I, II, and V), and the two Newton volumes published under the auspices of the Mathematical Association in England and the History of Science Society and other organizations in America. These, with sources already mentioned and with the helpful articles in the encyclopedias make possible a great deal of work that may be done without the necessity either of consulting a great historical library or of working in a foreign language. We can obtain the materials if we are willing to take the trouble, and if we do and use them, we as well as the students are the gainers.

THE SECTION-GROUP-INDIVIDUAL PLAN

B. Jeannette Riefling

Editor's Note: *Miss Jeannette Riefling is a successful and progressive teacher of mathematics in the Cleveland High School of St. Louis, Missouri. She is one of the relatively few teachers in any given system who are constantly striving to increase their own efficiency in the classroom by adjusting content and technique in an experimental manner to the needs and capacities of the pupils.*

J. A. D.

Curriculum revision has placed emphasis upon what to include in the content, but the technique to be employed in teaching this content is left almost entirely to the teacher. It may be argued that students have been classified according to some type of intelligence testing, or other method of sectioning; and that recommendations have been made as to what to include for each group. This, however, does not solve the problem of how best to present this material to the students.

Even in classes selected according to I.Q. there is great variation of ability with respect to mathematics. Within these sectioned classes further grouping for instruction and even individual instruction are found necessary to teach the work properly and to reduce failures to a minimum. This article will attempt to outline briefly actual procedures in giving a combination of group and individual methods of instruction to classes assigned during a specific term. The method is used in a public school where no special effort is made to limit teachers to special procedures, and is used independently of the methods used by other teachers of mathematics in that school.

Class I is a class in plane geometry. The work is essentially that of books one and two of any standard geometry text. As is customary, it is a class in the first half of the tenth year of school. The class is exceptionally strong and unified in its work. No sectioning has been done. A little individual help has been given before school in the morning. There are 32 in the class, 9 girls and 23 boys. The I.Q. is not known for three boys and three girls. The range in I.Q. is 110 to 144, the median being 120. The best student in the class has an I.Q. of 126 (fourth highest I.Q.). The next best has an I.Q. of 110. The poorest student has an I.Q. of 110. Second poorest has an I.Q. of 142. This rank is likely due to poor eyesight.

Class II is a class in solid geometry and comes in the first half of the eleventh year. Here again there is no sectioning. There are 27 in the class, 3 girls and 24 boys. One I.Q. is not known. The range is from 102 to 129, the median being 110. The best student in the class has an I.Q. of 109 and the second best an I.Q. of 118. The poorest students have I.Q.'s of 104 and 128.

Class III is a class of the same school grade as Class I and covers essentially the same work. This class, however, is a class of very low I.Q.'s or a "C" section instead of an "A" section as is Class I. There are 28 in the class, 6 girls and 22 boys, one boy having just entered the class in the past few days. His I.Q. is not known, and the others range from 80 to 102, with 93 as a median. The four best students in order, from first best, have I.Q.'s of 98, 94, 89, and 83, respectively. The four poorest starting with the poorest and going upward have I.Q.'s of 85, 91, 88, and 102, respectively. This class has been sectioned, the respective I.Q.'s of the superior group being: 84, 85, 87, 89, 89, 89, 93, 93, 94, 94, 99, 100, and of the poorer section: 80, 83, 85, 88, 90, 91, 93, 95, 95, 96, 97, 97, 97, 98, and 102.

Class IV is a class of first-term mathematics which is essentially an elementary-algebra course. There are thirty in the class, 8 girls and 22 boys. One I.Q. is not

known (that of a new girl). The range is from 93 to 118, the median being 103. The four best in order, based on achievement, have I.Q.'s of 105, 107, 101, and 98, respectively. The four poorest starting with the poorest, have I.Q.'s of 100, 107, 105, and 103, respectively. This class has been divided into four sections based on achievement:

Best section, I.Q.'s: 98, 100, 101, 105, 107.

Second best: 102, 103, 103, 105, 107.

Third best: 99, 100, 101, 102, 104, 104, 106, 106, 118.

Poorest section: 93, 99, 100, 100, 101, 103, 103, 104, 105, 107.

Class V is a plane-geometry class of the second half of the tenth year. The work consists of books three, four, and five of a standard plane geometry. There are 22 in the class, 3 girls and 19 boys. Three I.Q.'s are not known, the others range from 82 to 105 with a median of 95. The four best, in order, have I.Q.'s of ?, 91, 105, 96; the four poorest, starting with the poorest, are 99, 98, 94, and 98.

Better section, I.Q.'s: 82, 89, 90, 91, 94, 95, 96, 98, 99, 102, 105.

Poorer section: 89, 94, 94, 96, 97, 98, 99. Note that the median of the poorer section is slightly higher than that of the entire class.

Thus it is seen that in spite of grouping according to I.Q. the students of a class are likely to vary in ability to do mathematics. This is especially true in ninth-year classes as Class IV above shows. If students are weak in mathematics, it is well to give much help in class in order to avoid the formation of wrong responses in working alone at home. Formerly fifty-minute periods were used; but later, periods were shortened to forty minutes. No special division of the period has been made for testing or teaching, the needs of the moment determine this. It has been found that the great need for the weaker students has been adequate drill.

Depending upon the class in question, one of three methods is used:

1. Pairing students, one bright one with one weaker one, and letting the bright student supervise the work of the weaker one

2. Allowing each individual to go at his own rate, coming to the teacher's desk for help and to have work checked

3. Dividing the class into sections according to the rate at which each can work

Often combinations of these methods are used. In all cases the class is started at the same place in the regular work assigned to that term.

To show the working of methods 1 and 3 combined, the class described as IV above will be used. The work to be done will consist of:

1. Review of four fundamental processes with integers and simple fractions

2. Review of terms involved in 1, such as minuend product, etc.

3. Summary of all ways of expressing the processes in 1, both in words and in symbols

4. Meaning of positive and negative numbers, and the addition of such numbers

5. Definition of coefficient, the addition of binomials and polynomials, and the checking of addition by substituting small integers for the letters

6. Subtraction of positive and negative integers and monomials and checking by addition

7. Subtraction of polynomials and checking by addition

8. Definition of equation, the solution of simple equations by adding the same number or letter to both sides, and checking

9. Simple problems

10. Multiplication of monomials

After the class has covered the first five topics there is evidence of lagging on the

part of some students. A period is then spent with the students paired and working at the board. The teacher can move about the room giving suggestions on how best to teach the work needed, and in some instances actually to give help to the weaker students who are most in need of it. Towards the end of the period it is possible to separate the class into two groups, and also to select several students who show ability to teach others. A test is given to all the class, the papers graded immediately, and the class sectioned. In giving the assignment, the first group (the superior group) is given topic 6, while the other group is given topic 1.

The next day, group one is assigned written work on topic 6 while the other group recites on topic 1, and possibly topic 2. Then this group is started on topic 3. The work of group one is now inspected and help given as needed. Often some students need to be held back here for drill, though usually topic 7 is assigned to all the group. By the time this group is ready for sectioning the other group has usually reached the work of topics so that there is now likely to be a section on topic 5, one on topic 6, and one on topic 7. The personnel of these groups changes since some students work hard to get into upper sections, and others, by having some topic cleared up, automatically clear up other difficulties and move on to higher groups. Not all topics are equally difficult for all students, so a student who may be rapid on some topics, may slow up later on and drop back to another section.

Soon four groups appear with topics 5, 6, 7, and 8, respectively assigned. Those who are on topic 5 are given special help by the teacher and sometimes by student helpers and passed on to topic 6. It is usually necessary to maintain three or four groups in a beginning class with certain students getting individual help. When the work is such that all students are working to gain sufficient drill on a topic, all students are at their seats and working on paper. As soon as a certain unit is finished, or when help is needed, the student comes to the teacher. If help is needed, it is given and a new assignment of examples is made. If the work is properly done, new work is assigned. If the work is poorly done, the weak spots are pointed out and more of the same kind of content is assigned.

To prevent superficially rapid progress on the part of the capable students, two things are done: if some students are lagging badly, they are paired with the superior students; if not, then some difficult examples that test the ability of the best are assigned. They work in groups and often have very interesting debates over methods of procedure. Often the teacher is called to assist in settling some question. Where the method of pairing is used, the student helper is told what to attempt and he is responsible for his charge even to the assignments and testing. Often a student helper practises his own work on the board while the other student is doing some assigned examples.

The good students are enthusiastic over these methods of conducting classes for they feel that they not only progress as rapidly as they are able, but they get interesting review and a good understanding of the work that they teach. The weaker students like it because they get work at the rate best suited to them, they get much individual help, and in many cases they like the contact with other students better than contacts with the teacher.

Since examinations are required each five weeks, it is necessary for the teacher to prepare a test for each group. However, at the end of the term it is usually possible to give the same final examination since it

is customary in the school in question to excuse students with high averages from the finals.

By this method students who are absent or those who enter from other schools find that they have little difficulty in fitting into the class. It also provides motivation since the next higher group is the goal instead of the most advanced. In the case of the brightest the motivation comes through the additional instruction in how to locate difficulties, in how properly to question others and properly help them, and in thinking of work in terms of related units.

CALCULUS IN THE HIGH SCHOOL

John A. Swenson

Editor's Note: *Mr. Swenson is head of the department of mathematics in the Wadleigh High School, New York City. He has been a vigorous leader in the movement to reorganize high-school mathematics. Thousands of teachers have gone to Wadleigh High School to observe Mr. Swenson's instruction in the calculus. He is an instructor in the teaching of mathematics at Teachers College.*

J. R. C.

The report of the National Committee on Mathematical Requirements that appeared in 1922 gave four different plans for the reorganization of the mathematics courses in the senior high schools. In each of these four plans the recommendation for the 12th school year was calculus. Hence there is no doubt concerning what the Committee considered an ideal course in mathematics for the 12th year.

But so far, no general examining body has set a paper in high-school calculus, and up to the present time no satisfactory outlet or testing has existed to evaluate the experiment recommended by the Committee. However, many progressive secondary schools throughout the country have found out for themselves as individuals what high-school youngsters can do with a course in the calculus. Accounts of these experiments are given in The Reorganization of Mathematics in Secondary Education of the Committee published by the Houghton Mifflin Company.

In October, 1930, the mathematics syllabus committee appointed by the New York State Education Department met and recommended a course of study for the 12th year in harmony with the recommendations of the Committee. This means that the recommendations of the Committee will soon become more vital to the teachers of mathematics throughout New York State.

Many a youngster who has done very well in his high-school mathematics has found the college calculus a stumbling block. The reason for this is no doubt the attempted rigor on the part of the college teacher of calculus. When we realize that it took the foremost mathematical minds of the 18th and 19th centuries over 200 years to bring the calculus to its present-day rigor, it is no wonder that many a college youth finds this type of calculus beyond his understanding. Recently, however, some improvement in this respect has been brought about by introducing courses in mathematical analysis for the freshmen.

Hence one thing is sure, if the high-school teacher is to succeed with the calculus, he must not pattern his procedure after the college teacher. The calculus of the high school ought to bear somewhat the same relation to the college calculus that the intuitive geometry of the junior high school bears to the demonstrative geometry of the senior high school. That is, the high-school calculus ought to be an experimental study of variation and rate of change. At least, it should begin that way, and the more formal study of the calculus in the 12th year should be preceded by a graphical

study of variation and rate of change in the earlier years. Even as early as the 9th year, the graphing of straight lines should be done, in addition to the point method, by the slope-intercept method. Rates of variation should be obtained from graphs. The idea of slope would itself be a splendid introduction to numerical trigonometry of the 9th year.

In the 10th year, geometry should be studied from the standpoint of variation, and Cartesian methods introduced as a better preparation for the later work than the ancient geometry of Euclid. Some of the Euclidean methods may have to be retained but wherever an introduction of Cartesian methods will lead to a simplification of the present treatment, they should be brought in. Just because the Euclidean geometry has held sway for 2,000 years is no particular reason why it should do so forever. Then it would be possible to make a more complete and better study of loci in the 10th year than now. We could introduce the parabola, the ellipse, the hyperbola, in addition to the limited treatment we have now of the circle and the straight line.

In the 11th year, the functions should be studied from the standpoint of finite differences and also the converse problems attempted. That is, given a table or a graph, write the equation that fits the table or the graph. The converse problem ought, however, to be limited to functions of the first and the second degree, but the direct treatment ought to be extended to algebraic functions of higher degree and also to transcendental functions: exponential and trigonometric. This work in finite differences is a most valuable introduction to the calculus of infinitesimal differences of the 12th year. In this connection, we should always keep in mind that the readiness of the student for the calculus does not depend on whether he is in high school or college. By giving the youngster in the years preceding the 12th, a proper training in the function concept, variation, and locus, he may be better prepared to take up calculus in the 12th year than the present sophomore in college. It certainly must be admitted that the colleges have not made such an outstanding success of the teaching of the calculus that they are entitled to a monopoly in this line.

The new New York State syllabus for the 11th school year is essentially a combination of the former syllabi in intermediate algebra and trigonometry, with the recommendation that the function-concept be made the unifier. The proposed 12th-year syllabus is apparently intended eventually to replace the present syllabi in advanced algebra and solid geometry, the general scheme or main topics being essentially as follows:

1. Definite relationships such as the algebraic and transcendental functions with their application to equations and problems

2. Probable relationships, for example, statistics

3. Spatial relationships such as found in solid geometry

The calculus is to be the main tool in the study of these relationships. The details are now being worked out and the intention is to issue a very complete syllabus with copious notes for teachers. And here is also the opportunity for the teachers colleges to help and offer suitable courses that will prepare the teachers for this work. Let them offer not more pedagogy (we have had too much of that already), but courses in subject matter.

This new course will be truly a course in unified mathematics with all the various relationships, and not the present mechanistic course with one-half year of this followed by another half year of something else, apparently unrelated matter. If we do our duty to bring about this new type of mathe-

matics, we shall have nothing to fear from the so-called educational expert who, and perhaps rightly so, can see no value in the mathematics as taught at present. But let us remember that our mathematical salvation does not lie in cutting out all formal mathematics but rather in fitting the mathematical operations into the picture as they are needed in the application of mathematics.

As our civilization advances, the applications and need of mathematics continually increase. Formerly it was only the student of physical sciences that had need for mathematics, but now the students of social sciences are equally concerned. And the great difficulties that schools of education and business encounter in the teaching of statistics lie to a considerable extent in the poor preparation that the present type of high-school mathematics offers for such work. The emphasis in the teaching of mathematics must be shifted from the mechanical to the concept side or the thinking side of mathematics. This purpose will not be served by a more extended study of the old-type Euclidean geometry (whether plane or solid). The world had this type of mathematics for 2,000 years before the advent of the Cartesian geometry in 1637, but made practically no headway in mathematics. This point should not be overlooked in formulating our new objectives and courses of study.

PROVISIONS FOR SECURING AND MAINTAINING COMPUTING SKILLS IN THE FUNDAMENTAL OPERATIONS AS FOUND IN JUNIOR-HIGH-SCHOOL MATHEMATICS TEXTBOOKS FROM 1916 TO 1928

John A. Entz

Editor's Note: *Mr. John Entz was a graduate student in the School of Education of New York University two years ago. During this time he made a very intensive and most painstaking study of the content of sixteen three-book series of junior-high-school mathematics texts. The following paper is based on some of the findings of this study.*

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J. A. D.

Educators agree that arithmetic is studied in part for the purpose of gaining a knowledge of the four fundamental processes. These are recognized as the "machinery" of arithmetic. It is also held that the teaching of the fundamental operations of addition, subtraction, multiplication, and division with whole numbers, common fractions, and decimals, should be completed by the end of the sixth school year. This means that the pupils of this school level should not only have a fair knowledge of this phase of arithmetic, but also be able to perform these operations with speed and accuracy.

However, certain recent studies and examinations show that the schools are not securing the desired mastery at the end of the scheduled time. Practically all of the surveys lead to the same conclusion. Judging from the findings, it seems that there is abundant evidence to say that the junior-high-school pupils do not have that command of arithmetical computation which they should have. Some educators take the stand that the poor showing is due to the fact that the secondary schools have made inadequate provision for maintaining those computing skills which the elementary schools have labored to build up; that one of the functions of the junior high school is to provide for systematic reviews, so the material will be permanently retained, and that effective drill should be continued throughout the entire secondary-school period.

In order to determine just what the provisions are for securing and maintaining computing skills in the fundamental operations as found in junior-high-school mathematics textbooks, an examination was made. The data for the study were secured through the examination of sixteen three-book series for the seventh, eighth, and ninth grades. The exact number of examples for practice in the skills for all four fundamental processes with whole numbers, common fractions, and decimals, in each of the three junior-high grades were counted, and the number carefully listed under the proper caption. This study included only the examples intended for direct practice material for computation. The problem type was entirely omitted.

From the data obtained several interesting deductions may be made. The range is relatively wide, and the variability large. In the seventh year every textbook of the entire number makes definite provision for practice material in the four fundamental operations, both with whole numbers and with fractional numbers. The range for the examples of the seventh-year texts is from 382 to 2,266. Nine of the books have more than one thousand examples each. The eighth-year textbooks as a whole, furnish less drill material, although the range is from 0 to 2,323. Two texts contain no practice material. One text has fewer than one hundred examples, and these are found in the last chapter of the book in connection with the general review. In the ninth year of the series only three authors present a number of examples worthy of mention. The range is from 1,297 to 445. The remaining thirteen ninth-year textbooks have no provision for computational drill in arithmetic, or fewer than one hundred examples each. This means that the authors of these thirteen texts made no conscious effort to provide for such material.

There seems to be a difference of opinion in regard to the distribution of the drill exercises. Two different methods are employed. The majority of the texts concentrate this material in one or two chapters, while several apportion it throughout the book. The greater number of examples of this type is found in whole numbers and the least number in decimals.

Judging from the study it would seem that all the authors of the junior-high-school mathematics textbooks are agreed that some review of the four fundamental processes in arithmetic is necessary in the junior high school, and that some practice should be provided. The authors of the texts recognize that during the first six grades the pupil has learned to compute with some degree of skill, but that greater proficiency should be secured during the next few years. However, they do not agree on the amount or kind of practice.

If the median for the seventh year is the correct number, several texts contain too few examples, and two texts too many. If the eighth-year median is low, ten texts should provide more practice, assuming that to furnish adequate drill there should be from 800 to 1,000 examples. During the ninth year, the skills already acquired should be maintained. If we accept this statement, then at least thirteen ninth-year textbooks omit this phase of the work. Any new texts or revisions should take care of this feature.

The grand total of all the series shows the range from 382 to 4,451, with the median around 1,530. This would suggest that the highest one of the series may contain too much material, and surely the five lowest too little.

In conclusion then, it may be that further evaluation of this topic in junior-high-school mathematics should be made, and a fair and adequate amount of practice examples for the four fundamental processes in both whole numbers and fractional numbers be provided.

MAINTAINING A BALANCE BETWEEN SUBJECT MATTER AND METHODS IN TRAINING MATHEMATICS TEACHERS

J. O. Hassler

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J. R. C.

The two most important questions that confront high-school teachers are: What should we teach? and, How should we teach it? The tendency in recent times to change the emphasis from the first of these questions to the second has doubtless improved the teaching of mathematics. The important question to be considered now is how to balance the emphasis on the two aspects of teacher training.

We dismiss at once the views of the two extremists; one, the reactionary who clings to the view that teachers are born, not made, and that to know one's subject is to be able to teach it; the other, the radical educationist who has become so enthusiastic over the efficacy of the multitudinous and detailed theories of how we learn that he believes the only necessary qualification for a mathematics teacher, besides a knowledge of proper methods, is ability to solve all the problems in the textbook he uses. One of these extremists would have a bow with no cord—the other a cord with no bow. "As unto the bow the cord is" so is method to subject matter. We must have both if we teach the young idea to shoot. It is only a question of how much each should be emphasized.

In many teachers colleges and in some university schools of education the students are expected to take at least twenty per cent of the course for a bachelor's degree in educational methods, because that much credit will secure for the graduate a life certificate. (Why should beginners have "life" certificates when so many of them do not make teaching their life profession?) Additional courses are so strongly recommended that sometimes a third of the student's credits for graduation are in educational courses. At the same time the proponents of such extreme emphasis on methods consider that mathematics through one course in calculus is more subject matter than is needed in the preparation for the senior-high-school teacher—an equivalent of about fifteen semester hours of college mathematics (excluding high-school courses, such as solid geometry, that are taught by many colleges because so many students omitted them in high school). This would be about twelve per cent of the college course. Such a disproportionate emphasis is unjustifiable.

Why do some insist that a prospective teacher should master *all* the rules of his game before he plays it when there is no other analogous situation where such procedure is possible? No golf player ever played a successful first round because of reading a book of lessons on how to do it, nor by taking lessons from a professional, making practice strokes as he tried to learn. The early lessons help and should be had, but the most valuable ones are those taken after experience with the difficulties of the game. For the same reason a beginning teacher should have only a few fundamental courses in general psychology, adolescent psychology, history and philosophy of education, methods of teaching, and practice teaching. This furnishes him with a kit of simple tools he can use with proficiency.

As he gains experience and skill let him increase his knowledge of educational theory. It means much more to him then. Furthermore, he is not led to believe that he has completed his training, as is often the case when so many preliminary courses in methods are given. Many graduates of teachers colleges, after teaching for a few years, have expressed their dissatisfaction with such preparation and have insisted that all their work for a master's degree be in mathematics. They refuse to take more courses in educational methods and theories.

Wherein lies the values of a greater number of courses in college mathematics for a high-school teacher?

First, it gives confidence and makes it easy to dominate classroom procedure. What is more pitiful than the hesitating, uncertain teacher, who, without any background of wide experience in higher mathematics to give him a commanding grasp of his elementary subject, must refer to his answer book, or try vainly to recall how the exercise was worked out in the key he studied in preparation for the lesson? As his weakness is revealed to his pupils, they look on him with disdain and enjoy "stumping" him with hard problems. Consider on the other hand the teacher who "knows more than is in the book"; who does not blindly teach the errors in the text, but points them out to the pupils with such a clear, understandable explanation that the pupils admit the error and recognize his transcendent knowledge of things mathematical; who can lead his best pupils into higher realms of mathematical thought; and who thereby wins the approbation and good will of the class. Sometimes this helps more with discipline than a knowledge of psychology. It was said of the great penman, Spencer, that his class sat in rapt attention when he stepped to the board to set a copy, because of his consummate skill.

Second, it enables the teacher to choose wisely the best content for the course. He has had enough experience to know the nature of the applications of mathematics. He knows which of the mechanical manipulative processes are used so little even in higher mathematics (and never elsewhere) that they should not be required of a student. There are algebraic processes, now blindly taught in many of our schools by ignorant teachers under the assumption that they are preparing students for future mathematics, that I have never needed to use in years of study of higher mathematics.

Third, he finds many specific instances where his knowledge of the subject matter of higher mathematics will react favorably on his method of teaching. Every one can understand how a knowledge of analytic geometry gives a teacher command of the situation in teaching graphs. Similarly, courses in analysis lead to a clearer understanding of the function concept so vital in algebra. Courses in modern higher geometry, including non-Euclidean and projective, give a teacher a more wholesome viewpoint of the high-school plane geometry. The attempted display of knowledge by half-informed teachers concerning the concept of infinity has many a time caused confused thinking among students that could have been avoided if the teacher had known more mathematics.

Finally, as has been true in ages past, a scholarly teacher will inspire youth to higher ideals of learning—a half-informed teacher cannot.

Why not have at least as much college mathematics (not high-school mathematics in college) as educational methods?

WHAT THE SEVENTH-GRADE TEACHER MAY REASONABLY EXPECT FROM HER ENTERING PUPILS IN THE WAY OF ARITHMETIC ABILITY

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J. R. C.

Mathematics in the seventh grade can be made an interesting and profitable game if the teacher is willing to accept the pupils at the beginning of the year's work where they are rather than where the course of study says they ought to be, and to lead them as far and as rapidly as they can go profitably and joyously. Unless searching problem-solving and computing tests of the Stone and Cleveland Survey types are administered at the opening of the year rather than at the close of the sixth grade, the teacher is likely to waste much time and effort by using content which is much too difficult for the majority of the class.

From the upper third of the entering class the teacher may reasonably expect the habits, skills, and attitudes listed in this paper. What she finds in the lower third only a searching test covering fourth- and fifth-grade goals will reveal to her. Many pupils find themselves entering the seventh grade for reasons other than that they have met the requirements of sixth-grade arithmetic. Some are there because they are too old and too large for the sixth grade; others because there are empty seats in the seventh grade which must be filled; others because of the social and political prestige of the parents; and still others for reasons which only a perplexed and harassed administrator might reveal. In short, the well-informed arithmetic teacher finds herself facing a class sharply divided into two groups: (1) those who have actually done the arithmetic work of the sixth grade and have actually passed reasonably high standards at the end of the sixth grade; (2) those who are passed for other reasons.

In the upper third of the entering group the seventh-grade teacher may expect the following abilities:

1. In the fundamental facts of addition, subtraction, multiplication, and division, 99 to 100 per cent accuracy with a time limit of three minutes for each of the four tests.

2. In the fundamental processes with integers, ability to meet the standards required of sixth-grade pupils by the Courtis Research Tests, series B.

3. In common fractions

a) An understanding of the four concepts of a fraction

b) Ability to measure to sixteenths of an inch with a foot rule scaled to sixteenths

c) An understanding of the meaning of each of the four fundamental processes

d) At least 95 per cent ability in adding, subtracting, multiplying, and dividing two fractions such as $\frac{3}{4}$ and $\frac{2}{3}$

4. In decimal fractions

a) An understanding of the decimal fraction as a continuation of the integer scale to the right of the decimal point

b) An understanding of the significance of shifting the decimal point and some skill in shifting the point from left to right and from right to left

c) Good control of the decimal point in multiplication and division. Such control should come largely as a result of developing good judgment as to where the decimal point belongs in the product or quotient. Such judgment should be acquired by training in estimation rather than by obedience to a memorized rule.

5. In percentage

a) Knowledge that percentage is a special phase of decimal fractions in which the per cent sign (%) stands for hundredths

b) Some ability to express without written computation a decimal as a per cent, a per cent as a decimal, certain common fractions as per cents, and certain per cents as common fractions in lowest terms

c) Some judgment as to which of the foregoing skills is to be employed in an easy problem situation

d) Some ability to solve easy percentage problems involving either of these conditions: to find a given per cent of a given number, or to find what per cent one given number is of another given number

6. In problem solving

The seventh-grade teacher should realize that it is her particular job to give her pupils a real start in problem solving. In this important skill she is apt to expect too much even of the best in the class. How to attack problems, how to read problems analytically, how to use the equation, graph, or table after the problem is read, how to estimate a reasonable answer after the plan of the solution is formulated are elements in the solving of the two- or three-step problem about which sixth-grade children are almost totally ignorant.

In general, in the upper group the teacher may expect the right attitude towards arithmetic plus a desire to want to know more about mathematics. And this is probably the most encouraging factor in the difficult task confronting her. In other words, alertness to see number relationships, ability to see how numbers change together, problem-solving skill, the informational side of arithmetic are fields about which children have little knowledge and less skill when they enter the seventh grade. These are the fields of mathematics in which the first year of the junior high school should render important service.

THE PRESENT SITUATION IN MATHEMATICS IN NEW YORK CITY

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J. R. C.

In order to understand the present situation in mathematics in New York City, one must know the workings of the school system. While fundamentally the schools are organized on the 8-4 plan, the 6-3-3 grouping is gradually displacing it. According to the City Superintendent's report for the school year ending July 1929, there are in New York City 53 junior-high-school organizations, most of which are in conjunction with an elementary school, eight being in separate units. The senior high schools are all four-year organizations which receive pupils from the 8B grade into their first year and pupils from the 9B grade into their second year. The following table gives the distribution of pupils as of September 30, 1928.

Year	*8B Schools*	*Junior High Schools*	*First-Year Senior High Schools*
7	59,980	30,520	......
8	55,377	29,920	......
9	......	25,953	49,078

The three types of schools are not under the same supervision, and this makes for a lack of coördination which, it seems to

me, is one of the interfering factors in the success of mathematics in the schools.

The only subject, outside of English, which is continuous from the lowest grade to the highest is mathematics. Still, this fact is neglected as the pupil advances from one division of the system to another. In the 8B schools the pupils are given eight years of arithmetic. In the senior high school these pupils go on with algebra, geometry, and the advanced courses. In the junior high school the pupils, having come with six years of arithmetic, take two more years of arithmetic, followed by a year of algebra.

Recently, the course of study in arithmetic was revised as part of a larger curriculum-revision movement in the elementary schools. Concerning this, the district superintendent assigned to junior high schools says in his annual report:

> The adoption of new courses of study during the year for elementary schools gave rise to the question of using these courses in the seventh and eighth years of the junior high schools. The problem was discussed at meetings with principals. I have taken the liberty to recommend that the new courses be adapted to the work of this division. . . . . . .In the case of continuing subjects like English and mathematics, which were prepared without consideration of our ninth year, it will be necessary to make adaptations which will produce the necessary adjustment of subject matter.

Here is an illustration of the lack of coördination referred to above. The arithmetic committee contained nobody from the senior high schools. A sufficiently large number of elementary-school graduates continue mathematics in high school to make it necessary that the secondary-school point of view be taken into consideration in the teaching of arithmetic, especially since for years the complaint has been uttered time and again about poor preparation in arithmetic. The first assistants in mathematics took the initiative themselves and called the attention of the committee to a discrepancy between the City arithmetic syllabus and the State arithmetic syllabus upon which the ninth-year algebra course is based.

It is unfortunate that teachers of arithmetic consider themselves a body distinct from teachers of mathematics, as the expression is commonly used. They, therefore, pay little attention to situations which the children will meet in high-school mathematics; for example, (1) the use of the minus sign as a crutch in subtraction with an exercise like $\begin{array}{r}6\\ \underline{-4}\end{array}$ being read "four and how many are 6," in which the dash before the 4 is supposed to stand for the words "and how many are"; (2) the pigeonhole arrangement in the addition of fractions which must be abandoned entirely in algebra, and (3) the lack of understanding of the meaning of cancellation which to many pupils is the equivalent of drawing a line through a number. It seems to me that a number of notions that are developed in algebra could be started in a very elementary form in arithmetic, if the teachers were aware of the interrelation of the two subjects.

The fact that the junior-high-school problem in mathematics was disregarded is not so serious in my opinion, because the State Department of Education issued in 1928 a Tentative Syllabus in Junior High School Mathematics, which is considered the best of its kind that has been formulated up to the present.

A comparison of the State Junior High School syllabus with the new City arithmetic syllabus shows that there is no loss in the amount of arithmetic included. The arithmetic is presented in conjunction with informal geometry in the seventh and eighth years and with algebra in the eighth and ninth years. The point of view is well expressed in the following quotation from the syllabus itself.

If I had the authority to do so, I would have this syllabus introduced into all sev-

enth and eighth years, whether in 8B schools or in junior high schools. If this is the better mathematics—and it undoubtedly is—then all pupils should derive the benefit of this course, no matter where they are. The first assistants in mathematics in the senior high schools have even placed themselves on record as favoring the adoption, for the first year of the senior high school, of the ninth-year course of the junior-high-school syllabus as a substitute for the present elementary algebra, provided the graduates of the 8B schools have had the prerequisites in the seventh and eighth years. No action, however, was taken.

This in brief is the situation in mathematics in the elementary schools and in the junior high schools. There is no reason why an artificial distinction should be continued between the seventh and eighth years in one building and those in another, when the type of organization in which a pupil finds himself is purely a matter of chance. The remedy, it seems to me, lies in the appointment of a director of mathematics with complete authority over the mathematics of all divisions of the school system from the kindergarten to the last year of the training school. This would take the matter from the general educators in the schools, whose major interest after all is administration and supervision, and place it in the hands of the subject-matter specialist where it should be—the specialist who is not merely a scholar in mathematics or not merely interested in supervision, but who is also thoroughly acquainted with the best practices in connection with mathematics curricula and in classroom procedure.

The first problem that one meets in the senior high schools is the omission of algebra from the first-term program in a number of schools. The correction of this weakness, we are told, rests with the principals. It is a matter of common sense, it seems to me, not to allow pupils to omit one term in a continuous subject like mathematics.

The course at present consists of one year of elementary algebra, one year of plane geometry, and one term each of intermediate algebra, advanced algebra, solid geometry, and plane trigonometry. The following table gives the distribution of pupils in these grades as of January and June 1929.

	January 1929		*June* 1929	
	No. of Pupils	Per Cent	No. of Pupils	Per Cent
Elementary Algebra...	25,322	43.0	27,489	43.2
Plane Geometry........	23,682	40.0	24,540	38.6
Intermediate Algebra...	6,747	12.0	7,652	12.0
Trigonometry..........	1,864	3.1	2,290	3.6
Advanced Algebra......	821	1.5	1,055	1.7
Solid Geometry........	617	1.4	562	.9
Total..............	59,953	......	63,588	....

In addition, there were also 11,742 pupils in January and 11,389 in June who, according to the superintendent's report, took special courses in mathematics, the nature of which is not described in the report. The difference between the 40 per cent and the 38.6 per cent in the case of plane geometry is due to the fact that special classes in geometry for slow pupils are not included in this table.

At present the mathematics curriculum is in a state of flux. This is part of the general curriculum-revision movement that has swept the country. Since the Report of the National Committee on Mathematical Requirements appeared in 1923, the tendency has grown to revise the courses in accordance with the progressive ideas of the leaders in the field.

The New York State Mathematics Syllabus Committee, following the spirit of the national committee report, has issued a new ninth-year syllabus which includes a unit of numerical trigonometry to help the pupils to get a broader view of the field of elementary mathematics. This is not all that can be done, but it is the best under the circumstances. The three-year junior-high-school syllabus, referred to above, is the best, and that is why the first assistants in

mathematics are willing to make the substitution described.

The question is frequently asked as to what extent emphasis in algebra is placed today on thinking as opposed to the purely mechanical operations. This may be answered by a study of Regents examination papers in elementary algebra, on the theory that the teaching in the classroom is colored by the make-up of the papers. A cursory examination of the papers of the past ten years leads one to see that the number of problems has been increasing slightly and that the number of so-called "power" questions has increased to a greater extent. It is not possible for a pupil today to obtain a passing mark and, at the same time, to avoid both the problems and the "power" questions. In fairness to the teachers it must be said that more time is actually spent on problems by the better teachers than the papers would indicate. I base this statement on my own experience with teachers and on conversations with my colleagues.

The State committee has also issued, for voluntary adoption, a tentative syllabus for the mathematics of the eleventh year, which combines algebra with plane trigonometry. The purpose in this is to have comprehensive courses in mathematics rather than the separate one-term courses that are ordinarily found in the high schools beyond the tenth year. While the new course merely lists the topics and does not indicate any teaching sequence, an experiment with it, conducted in the George Washington, the James Monroe, and the Thomas Jefferson High Schools, favors the fusion of trigonometry with algebra.

There is now in the course of preparation a new twelfth-year syllabus which also embodies the spirit of a comprehensive one-year course in mathematics as a sequel to all that has preceded.

Since the present one-term courses will continue in force, they are also being revised to conform to the new spirit. And so, in the intermediate algebra course much greater emphasis is placed on the formula, the graph, and functional thinking; and the advanced algebra course will include a unit on differentiation in connection with the application of the graph in problems of maxima and minima, and also a unit of elementary statistics.

In geometry the tendency has been away from the *memoriter* book work towards power in doing original exercises. The number of propositions for examination purposes has been reduced to a minimum, and, wherever possible in the year's work, they are also to be treated as exercises. This is in line with the present trend in the Regents examination in plane geometry.

In two of the City high schools a modified course in geometry is being taught to the weaker pupils who are segregated into separate classes at the beginning of the year. In the Manual Training High School the basis for classification is an intelligence test and the achievement of the pupils in algebra; in the George Washington High School it is the Orleans Geometry Prognosis Test. In each case the outline of the course was prepared by the chairman of the department. At the end of the year the pupils receive school credit only and are not prepared to take the Regents examination. If this plan is successful in two schools, it should be adopted for the others, where the problem of failure in geometry is just as acute.

The question of a combined course in plane and solid geometry, which is now being discussed throughout the country, is under consideration by a committee of first assistants. Perhaps this will be the form which the revision of the tenth-year work will take.

The matter of a two-year comprehensive course in general mathematics for the ninth and tenth years is also being considered by

a committee of chairmen. The plan is perhaps to ask for the approval for such a course for those schools that may wish to introduce it. The pioneer in this movement in New York is Mr. John Swenson of the Wadleigh High School, where, for a number of years, he worked with a comprehensive course in elementary mathematics. As the complexion of his school changed, he had to abandon the work gradually, but he is still the leader in the movement to have such a course offered officially for voluntary adoption in the City. An unsuccessful attempt was made to experiment with the general mathematics in one of the other high schools. Its failure was due in large measure to the lack of support by the teachers.

In treating any important school problem one must of necessity deal with the eternal school triangle—the subject, the teacher, and the pupil. What I have said so far concerns particularly the subject matter. A word now about the teacher. That the teacher is an important factor in the situation goes without saying. At times, she is an interfering factor, as, for example, the teacher who fails to get from her pupils the type of work they are capable of producing. The attitude of the teacher and her preparation are important elements in the success of all curriculum revision. When a change in teaching procedure, or in content, or in sequence of topics is proposed, the question is immediately raised as to whether the teacher can be depended upon to carry out the proposed change. For, after all, a course of study is best evaluated through experiment in the classroom and not through discussion in committee meetings. To what extent the teachers of mathematics of New York City are well prepared or poorly prepared I am not in a position to say without a great deal of investigation; but my own experience with teachers and the remarks of other chairmen lead me to feel that, in the recently instituted professional alertness movement, teachers of mathematics should be required to take courses that will help to improve their classroom procedure, or will give them an acquaintance with the modern movement in the field so that they will understand the reasons for the changes in which they are taking part. For this reason I also feel that the courses in methods for teachers of mathematics in the local institutions should be given by men and women who are actively engaged in the work. Just because a person once taught mathematics, he is not necessarily fitted to train teachers in a highly specialized classroom practice, especially when that person's major interest is quite evidently in another sphere.

The part that the pupil plays in the matter under discussion is pictured in the annual reports of the City superintendent of schools. The following quotations are taken from the reports for the years 1925, 1926, 1928, and 1929:

> Our most serious problem has been the mathematics departments, where the results, as tested by the Regents Examinations, have not shown as high a standard as is shown in other subjects.....
>
> I dare venture the prophecy that the results would show that thousands of pupils are not gaining from the study of mathematics in the first two years of the high school sufficient benefit to warrant the cost to the taxpayers of such instruction ...... Fewer pupils should be encouraged to elect geometry in the second year.
>
> If we measure the difficulty of passing a subject by the percentage of pupils who fail, then mathematics is the most difficult subject. Between 77 and 78 per cent of the pupils taking courses in mathematics were successful. . . The percentages of pupils who complete successfully the work in mathematics are lower than the percentages in English. In fact the per cent who pass in the first term of elementary algebra is the lowest for any subject given in the high schools. No term's work in any subject has as high a mortality. The results in the first term of plane geometry are almost equally low. The results in the first term in elementary algebra and in plane geometry show

an apparent need of prognostic tests in mathematics or some other means of preventing a large number, who are certain not to pass, from taking the subject. . . . . . We note that in two schools more than 50 per cent of the pupils who took first-term algebra failed in it. At the same time two schools reported every pupil as having passed. Although these differences can be explained partially by differences in the character of the school population taking work in mathematics, they are probably also due to differences in standards

In January 1929, only 75.4 per cent of the pupils in mathematics were successful and in June only 73.6 per cent. . . . . . First-term algebra appears to have the highest mortality of all subjects. For the City as a whole only 68.3 per cent of the pupils in this subject succeeded in passing in January 1929 and only 65.6 per cent in June 1929. In one school more than one half of the pupils failed first-term algebra, while at the same time every pupil passed the subject in another high school. Although we may attribute this situation in part to the difference in the type of pupil, yet we are forced to the belief that even in so exact a science as mathematics there are variable standards of promotion.

The situation in elementary algebra and in plane geometry is serious; but, in view of the changes that have been introduced, the fault lies neither with the subject matter, nor with the pupils, nor with the teachers. The mass of classroom teachers of mathematics seem to accept things as they are. They are good followers. They ask no questions and do the work assigned to them. They have not taken the initiative in making changes, perhaps because they have felt that they have not the power to alter conditions. The fault lies with the chairmen who have accepted year after year the description of the poor work done by the pupils and the criticism of inefficient teaching. They should have taken the challenge and either brought about improvement or suggested remedial action. The summary for the term ending June 1930, judging from the reports submitted by the chairmen, will be no better than the summaries for preceding terms. Furthermore, the distribution of final marks by step intervals of ten paints a picture that is still less bright.

Per cent of pupils receiving class marks in algebra and geometry between 90-100, 80-89, 70-79, *etc., for the term ending June* 1930 *in the high schools of New York City.*

Intervals	*Alg.* 1	*Alg.* 2	*Geom.* 1	*Geom.* 2
90-100	7	9	8	8
80- 89	14	16	16	17
70- 79	22	25	24	30
65- 69	24	27	26	28
60- 64	3	2	3	3
50- 59	13	10	12	7
40- 49	9	7	6	4
Below 40	8	4	5	3

If the class marks are an index of what pupils can do in algebra and in geometry, then about 25 per cent of the group may be considered superior. The 70-79 group contains a large number of pupils whose mark is just 70. The summary does not tell how many, but I was able to obtain this information from two of the large schools which may be taken as representative of all the schools.

Per cent of pupils receiving a final mark of 65 *or* 70 *in algebra and geometry in the George Washington High School and the Thomas Jefferson High School, New York City, for the term ending June* 1930.

	Alg. 1		*Alg.* 2		*Geom.* 1		*Geom.* 2	
	65	70	65	70	65	70	65	70
Per cent of total register	24	16	30	26	21	24	25	22
Per cent of passing marks only	36	24	35	32	30	32	30	29

This means that approximately 50 per cent of all the marks and from 60 to 65 per cent of the passing marks show borderline achievement. What then have the pupils carried away from their elementary mathematics?

There is one solution that comes immediately to the mind of every person who has had any classroom experience at all. If secondary education is to be really universal

and if the experiment being conducted in the United States for the first time in the history of the world is to be true democracy in education, then each pupil should be given a real chance to develop according to his ability. The cry that the gifted pupil and the dull one benefit from contact with one another is good theory. But what happens in practice? Here is a teacher with five classes totaling 180 or more pupils, some of the classes having 40 or more pupils in rooms provided with 35 seats; with three grades of work to prepare every day, for whom she must do clerical work of various kinds; in whom she is expected to arouse school spirit and a proper attitude towards school in general; to whom she must sell membership in the school organization, tickets for various school events, and subscriptions to school publications; and with an extra assignment or two, or more, in the study hall, in the lunchroom, or in the corridor. In each of her classes the range of ability spreads from the dull to the gifted. Should she key up her work to meet the best or should she slow up for the sake of the weakest ones? As a human being she will give more time to the poorer children than she can afford and her standard is, therefore, set for the average. The gifted ones and even those who are only above the average do not work to their capacity. Neither are the dull members of the class deriving full benefit from their work, because the content of the course is more than they can digest and the time in which it is to be covered is for them all too short. The result is the high percentage of failure that is found in the tables. The only solution for the problem of failure in elementary mathematics is the segregation of the poor pupils into separate classes and the introduction of City-wide modified courses of study which will carry with them school credit, but not Regents credit. And since weakness in algebra is due to a certain extent to a poor foundation in arithmetic, this modified course in algebra should give the pupil the necessary preparation which he did not get in the lower grades. It would be possible also for a pupil whose work indicates that he is out of place to be transferred to the proper group. The parents and the students should have very little to say about this. We are the experts in the field of education, and if we have the data to support our decision, we should stand by what we do. The best technique in the handling of normal and gifted pupils is to remove from them those who serve as a drain on the class and on the teacher without deriving any benefit themselves. From the point of view of the administrator this is entirely possible. In the large city high schools it is just as easy to program twelve grades of a subject as eight. I say this as a result of ten years of experience in the making of the program of a large city high school in which I scheduled with ease eighteen grades of English, twelve grades of mathematics, ten grades of Latin, ten of history, twelve each of French and Spanish, ten of music, and more than twenty of art.

Another solution of the problem might be to give the weakest pupils a very elementary type of non-algebraic mathematics in which they can succeed. Mathematics plays such an important part in the child's surroundings that he should become acquainted with its fundamental notions even in a diluted form.

One might also suggest that the introduction of the Dalton plan or of a modified project method which breaks away from the traditional class recitation towards the individual treatment of pupils would provide for varying abilities. This is also good theory. Can we expect the teacher working under the conditions described above to be responsible every day for close to two hundred pupils as individuals?

This is the situation in mathematics in the New York City high schools. Nobody is more aware of conditions than the first assistants themselves. For the past three years they have been meeting four or five times a year and have been trying to bring about improvement, but progress is made slowly. They can only suggest changes and remedial action. They are now represented on the State Syllabus Committee and are exerting the proper influence in formulating progressive changes in the curriculum. They are working on modified courses in elementary algebra and in geometry that will be suited to the ability of the weak pupils. They sponsored the organization of the Association of Teachers of Mathematics from both the junior and the senior high schools, in order to make the classroom teacher conscious of the present-day movements in the teaching of mathematics and to bring about the proper understanding between the teachers of the junior high schools and those of the senior high schools. There is still a great deal to be done. Merely to point out that the percentage of failure is too high is not sufficient. A remedy must be found and applied. To summarize:

1. Lack of coördination in mathematics among the three divisions of the school system can easily be remedied by limiting the arithmetic course to the first six years in the elementary school, and by introducing the State junior-high-school syllabus into the seventh, eighth, and ninth years, no matter in what type of organization they may be.

2. Some type of mathematics should be included in the first-term program of every pupil who plans to take any mathematics at all in the high school.

3. Mathematics concerns such a large part of the school population and so many years of each child's school career that there is need for a subject-matter expert to direct the work throughout the twelve grades, and also possibly the four years of the training schools.

4. Teachers of mathematics, if their supervisors think it necessary, should be required to take courses that will improve their classroom procedure, instead of registering for anything that they may find convenient in the college catalogues.

5. The poorest pupils should be grouped by themselves and be given a modified course prepared by the subject-matter experts and approved by the authorities.

6. Steps should be taken to determine the best method of locating and classifying the weak pupils. Various studies have been made concerning the use of intelligence tests, prognosis tests, and elementary-school ratings as a basis for prediction of success in high-school work. Sufficient data are available for a thorough study of the problem as it concerns New York City. The time-honored method of permitting the children to struggle with the subject matter for a term or more and to develop the unnecessary and unfortunate habit of failure should be abandoned.

SOME PROBLEMS OF THE NINTH-GRADE MATHEMATICS TEACHERS

ROBERT E. FADDIS AND J. ANDREW DRUSHEL

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In 1880 the high-school enrollment in this country was 110,000; in 1890 it was 200,000; in 1900, 520,000; 1910, 915,000; in 1920, 2,200,000; in 1930, 4,000,000 (estimated). It will be noted that beginning with 1880 the high-school enrollment has doubled for each decade for the last fifty years. The population has not doubled each decade by any means.

This rapid growth of our high-school population is in accordance with the American philosophy of education that "whosoever will may come." This is a real democratic spirit. No one is barred. All are encouraged to begin high school and many who would not go are sometimes made to go.

As a result of this philosophy the quality of pupils now entering high school has changed in the direction of poorer selection. They vary widely in experiences and interests and also in inborn abilities. On this account the median intelligence of our present high-school population must of necessity be much lower than that of the high-school population of 1890, although it is recognized that we have no actual measure of the intelligence of the 1890 group.

The immediate consequence of this philosophy for the last 20 years has been a much more rapid growth in high-school attendance than curriculum growth in the way of adjustment to the needs of such a widely differing school population.

What has all this to do with the topic under consideration?

Until quite recently algebra was a required study of all 9th-year pupils and still is for 90 per cent of our present entering 9th-grade pupils in spite of the fact that less than 50 per cent are capable of profiting by the type of algebra usually found in freshman courses. Out of this practice we have our present high mortality (sometimes as high as 40 per cent), a poor preparation and a distaste for algebra on the part of most of those who are passed (observe the statement "are passed," not pass), dissatisfaction among the earnest and conscientious teachers (and we still have many such) in 9th-, 10th-, and 11th-grade mathematics, not to mention the teachers of college mathematics.

At the present time about 50 per cent of the pupils who pass first-year algebra lack mastery of its very simple elements when they come to the 11th grade, shown by several studies made between 1920 and 1930. A little later reference in greater detail will be made to three such studies made within the last eighteen months. Algebra as a tool has no value for pupils who are not secure in its technique. Algebra has no disciplinary value when pupils have only 25 per cent efficiency. In passing, it is suggested that the relative amount of formal work in the mechanics of elementary algebra (that is drill) can be reduced enormously if we teach for mastery through understanding rather than for mastery through habituation without understanding, and if we eliminate those processes and operations seldom, if ever, used in later work.

How well do 11th-year students in high-school physics know the elementary algebra needed for all the problems, formulas, figures, and discussions required in a one-year physics course? From a study of five representative texts in physics by reading every line, studying every formula and figure, and

solving every problem, a total of 66 different arithmetical and algebraic knowledges and skills was found. This total constitutes Part I of the Kilzer-Kirby *Inventory Test for the Mathematics of High School Physics.*

Recently this test was given to an 11th-grade physics class of 77 boys in St. Louis University High School, also to an 11th-grade physics class of 45 boys and girls in a small city high school of New Jersey, and later to another 11th-grade physics class of 46 boys and girls of the same high school. All these students had one year of elementary algebra in the 9th grade and one year of plane geometry in the 10th grade.

The test was given to the first group of Millburn students, 45 in number, at the beginning of the second semester, February 1930, and to the second group, consisting of 46 students, at the beginning of the school year, September 1930. The St. Louis group consisted of 77 students, and the results of the test were published in the January 1930 number of *School Science and Mathematics.* All the students tested were of the 11th year, having had first-year algebra and plane geometry. It should be kept in mind during this study that algebra was given in the 9th year, and plane geometry in the 10th year, as it may bring some special significance to bear on conclusions to be arrived at after an examination of the results.

First it might prove interesting to examine some of the problems which were failed the greatest number of times by the three groups in Part I of the test. These problems should also give some idea concerning the nature of the test as a whole. The numbers listed to the right of the problems indicate the number of wrong answers for each respective problem. The figures in parentheses are some wrong answers that were given.

Example Number in the Test	*Examples*	*Class of* 45 *February,* 1930	*Class of* 46 *September,* 1930	*Class of* 77 *September,* 1929
9	Divide ab by a.	2	8	16
11	Add: 3c and —8c.	7 (—11c, —5)	6	7
15	If D=10, h=15, and H=45, what is the value of d in this equation: $D=\frac{h}{d\,H}$	6 (3 1/3, 300)	9	33
20	Divide —27d by —9.	2 (—3d)	12	30
21	The altitude of a rectangle is 2 ft., and the base is 6 ft. Find the area in sq. ft.	8 (6, 18, 144)	9	25
22	Square 3/8.	8	3	29
34	Find the value of 3/4x8/9x10/15. Give answer in lowest terms.	13	15	26
40	Expand: $(m-n)^2$	12 (m^2-MN-N^2), (M^2-N^2)	17	54
42	If 5% of a number is 10, find the number.	14 (50, 20)	15	29
43	Change .045 to per cent.	11	16	48
44	The radius of a circle is 2 inches. What is the formula for finding the area?	8 (DxPie, RC)	20	52
46	Change 1/9 to decimal form.	13 (.009, 11.11)	30	56
47	How many cubic in. are there in 1 cubic ft.?	15 (27, 144, 3)	22	54
49	$S=\frac{1}{2}at^2$. If S=10 and a=5, find the value of t.	9	23	62

Such answers as $\frac{1}{8}$, 8, and 18 were given for the value of 1^8. Many thought $\frac{3}{4}\%$ was the proper answer to "change .75 to per cent." The common mistake for 40^2 was 160. In "$3a+4a=?$" many said $7a^2$.

An examination of these results indicates one of two things. Either there was a tremendous loss of easy arithmetic and algebra skills during the 10th grade or many pupils were passed out of the 9th grade lacking such skills in a large degree. In either case the situation is a challenge to teachers of elementary algebra.

Recent studies in problem solving indicate that pupils at the close of their elementary-algebra course show themselves incapable of solving easy 8th-grade arithmetic problems or 9th-grade algebra problems. It is the exceptional students, possibly the top tenth, who acquire fair problem solving ability by the end of the 9th grade. Students fail because of inability to read analytically, because of poor methods of problem attack, because of lack in the fundamental arithmetic skills, and because of lack of number sense if by number sense is meant ability and alertness to see number relations.

In general, algebra computing and reasoning-test results indicate that many pupils are now taking algebra who should not be permitted to do so. Their presence in the class is detrimental to those who can profit from such a course because the tendency is to teach down to the level of the lower third hoping to get some of this group through the regents' examination. In the meantime the better ones are loafing, if not sleeping, on the job.

Doubtless from the foregoing it is discovered that the conscientious and earnest teacher of first-year algebra has numerous and difficult professional problems confronting her all through the year's work. These problems lie in three fields; namely, pupil classification, determination of specific goals, and selection of proper content for the realization of the goals set.

The important goals to be attained by pupils before entering 10th-year mathematics are:

1. A proper learning attitude towards mathematics
2. A correct technique in the four fundamental operations in arithmetic and algebra
3. Arithmetic and algebra sense
4. Some ability to generalize
5. Some ability with formulas, graphs, equations, and tables
6. Some skill with the function concept in problem solving.

MATHEMATICS IN PROGRESSIVE SECONDARY SCHOOLS

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J. R. C.

It is the object of this article to discuss some of the distinctive features of the mathematics program of the progressive secondary schools of today. At the same time, I hope to dispel the erroneous notion, held by some people not thoroughly familiar with the situation, that the mathematics content of the courses in such schools is lacking in material which challenges strenuous mental effort, and that the instruction given fails to place due emphasis on a mastery of facts. While this discussion pertains primarily to mathematics as taught in the John Burroughs School, the facts are equally true in the case of the majority of the progressive schools.

The mathematics curriculum of such schools consists of a full six-year program of rich content. The first three years are devoted to a study of that arithmetic, geometric, and algebraic material which best stimulates and sustains the pupil's interest, which contributes to his understanding of the facts about him, and which enlarges his mathematical horizon. The last three years are devoted to a study of plane geometry, algebra, trigonometry, and the fundamental ideas of analytical geometry, differential and integral calculus. Emphasis throughout the course is placed on the use of good sense and good judgment in arriving at desired ends. The course is enriched through the use of a surveyor's transit in both the junior and senior schools, by means of a study and use of the slide rule, through exhibits of mathematical instruments and calculating devices, and by means of an extended study of the history of mathematics. This latter study often leads to the writing of a brief history by a group of students on some topic which they have carefully investigated. That the course is duly rigid is attested to by the fact that students pursuing such a course successfully pass the College Board Examinations in the various subjects studied. Moreover, these students frequently continue the study of mathematics in college.

Teachers of mathematics in such schools believe, too, that they are obligated to become thoroughly familiar with each pupil and with the type of work he has done and is doing now in that subject, in order to advise him wisely concerning the continuation of the study of mathematics. During the ninth school year, therefore, abundant opportunity is offered each student to learn the nature of the mathematics work ahead. References are made to advanced subjects to stimulate interest and to arouse a desire to learn more about this important science. Pupils are encouraged to scan books on advanced mathematical subjects which are placed in conspicuous places to be delved into by those of a curious turn of mind. Pupils who in the face of these facts approach the end of their ninth-grade work with little or no interest in mathematics, with poor achievement, and with minds of poor quality, are definitely told that it does not seem advisable for them to continue work in this field. In this way, those pupils continue the study of mathematics in the senior school who have shown ability and evinced an interest in it. With such a group, a happy situation is brought about and an extensive and intensive treatment of senior high-school mathematics made possible.

In the progressive school, the child is the unit, and, therefore, no method of instruction is good which fails to properly stimulate the growth and understanding of each pupil. To attain this end, the work of each subject is laid out in units covering a related portion of work. A time is specified within which the work of the unit must be completed. Until that time arrives, each pupil plans his work and attacks it with whatever zest he wishes. The class recitation is very frequently omitted. The pupil uses most of the class periods in working under the supervision of the teacher, thus learning how to begin work promptly, how to read for information, how to find source material, how to avoid distractions, and so on.

Since the abilities of the pupils differ, they progress through the work of the unit at different rates. Minimum essentials are set up for all, while the more able pupil investigates some additional topic in which he finds interest. In this way, the bright pupil is encouraged and given an opportunity to work to his fullest capacity, while the very slow pupil is enabled to master the essential facts of the unit.

At the close of the unit the class is brought together for review purposes. Class recitations are conducted, general principles stressed, and particular difficulties discussed. Reports on extra projects are also included in such reviews, thus encouraging the individual having made the investigation and stimulating the minds of the other pupils.

The method of instruction also includes a testing program, the major purpose of which is to improve instruction. Tests, consisting of many brief questions, are devised for a unit of work. Answers are provided for the questions on such tests so that the pupil can immediately learn what weaknesses, if any, exist. In this manner a weakness is detected before a wrong habit can be fixed, and a situation is created for the most effective learning of more mathematics.

Finally, the progressive mathematics department is always considering wherein it can coöperate more closely with the various other high-school departments, and where it can use them to advantage in attaining the desired ends. Accordingly, the equipment of the physics laboratory is always employed when studying the parallelogram of forces in plane geometry, direct and inverse variation in second-year algebra, and forces and vectors in trigonometry. Frequently the physics teacher lends his assistance in these matters in the preparation of demonstrations and the explanation of principles. The mathematics department lends its assistance to the physics department wherever possible, especially in stressing mathematical processes that find frequent use in physics.

Coöperation with the language departments consists in calling into use what Latin the pupils know when explaining the origin and meaning of certain mathematical terms. In English, the coöperation consists in requiring of all mathematics students neat papers, exact sentence structure, correct spelling, and the use of other grammatical principles which the pupils are supposed to possess. Further coöperation consists in discussing with or reporting to the members of the English department pupils with definite weaknesses that can be corrected by that department.

The mathematics department further correlated its work with the other departments by availing itself of the facilities of the dramatic department, the fine-arts department, and the industrial-arts department. Frequently, an historical incident is dramatized by the aid of the dramatic department. Students go to the fine-arts department to model the heads of famous mathematicians, to construct from clay an Athenian school, or to model a Greek or Roman structure. The industrial-arts department is used by pupils for constructing such instruments as ancient levels, angle bisectors, parallel rulers, a surveyor's transit, the figures of solid geometry, and so forth. An interest in drawing or design aroused in mathematics is reported to the industrial-arts department with the result that the pupil pursues with interest a course in architectural drawing, the construction of house plans, cabinet designing, or other phases of drawing.

The mathematics department of the progressive school is seeking to make its course of study as rich and varied as possible, yet at the same time emphasizing thoroughness and exactness on the part of the pupil. It is correlating with other high-school subjects, wherever possible, in an attempt to make the whole of high-school education more meaningful to the pupil.

RECENT DEVELOPMENTS IN THE TEACHING OF GEOMETRY

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J. R. C.

During the last thirty years a marked change has taken place in the purposes of secondary education. The various changes in the social and industrial life of the people, the changing character of the secondary-school population, and the spirit of the new age have brought the secondary school face to face with new problems. Recent advances in educational psychology have revolutionized our conception of the educative process. The new philosophy of education has emphasized the changing character of our civilization and has demanded an education that is not static but dynamic. The function of education is not merely to teach the specific subject matter in order to preserve the achievement of the past, but also to prepare the pupils to meet the problems of a constantly changing social order.

The secondary school is moving forward to meet the new social demands. The objectives of the various courses of study have been altered or modified in harmony with the new educational aims. Less emphasis is being placed upon storing the mind with facts of information and with literary quotations, and more on the development of habits, attitudes, appreciations, and powers.

Probably no other subject has undergone a greater change in this respect than geometry. There is a common notion that geometry is static; and in a certain sense this is true. The facts and principles of geometry is static; and in a certain sense this methods; that when they are once proved they become unalterable truth, the same yesterday, today, and forever. But geometry as an instrument for educating citizens is as dynamic as any other school subject. Some idea of the recent progress in the teaching of geometry may be obtained by comparing the course taught thirty years ago with the present course. Only a few of the more important changes can be noted in this paper.

1. *Intuitive Geometry.* One of the most important changes in the teaching of geometry during the present century is the differentiation between intuitive and demonstrative geometry, and the teaching of intuitive geometry in the junior high school. Three decades ago instruction in intuitive geometry was largely limited to the mechanical mensuration taught in connection with the traditional arithmetic. The growth of the junior high school with its broad outlook and exploratory function offered an opportunity for a survey course in mathematics, including geometry. The introduction of intuitive geometry into the curriculum before algebra is in harmony with the psychological principle that instruction should proceed from the concrete to the abstract. It is also in accord with the historical development of geometry and algebra by the race. The work in intuitive geometry is being organized so as to form a gradual approach to, and provide a foundation for, the work in demonstrative geometry. The next logical step is to introduce intuitive geometry to the lower grades so that the child's concepts of number and space may be developed side by side.

2. *The Introduction to Geometry.* The traditional approach to geometry was by way of memorizing six or eight pages of formal definitions, axioms, and postulates whose meaning the pupil did not under-

stand, and of which no immediate use was seen or made. Having memorized these abstract statements, the pupil was suddenly plunged into the formal logic involved in geometric demonstration, without having learned the meaning of a proof or seen the need for one. The result was too often lack of interest at the outset, followed by early discouragement and later failure.

During the first decade of the present century, leading teachers began to agitate for reform. The formal introduction of definitions has been replaced by an exceedingly interesting chapter of from thirty to fifty pages. The modern introduction points out the uses of geometry and its applications and emphasizes the value of its study in terms that the pupil can understand. It contains many pictures illustrating geometric forms and designs in nature, art, and industry which serve to create an appreciation of the beauties of geometry and to maintain interest in its study. One of the important features of the present course in geometry, which was entirely lacking in the course of thirty years ago, is the use of instruments in drawing figures and designs and field measurements. The use of the instruments of construction helps the pupil to acquire the vocabulary of geometry and to understand many geometric concepts and relations while he is still working with concrete things.

The memorization of definitions in the traditional introduction gave pupils the impression that geometry was a subject for mere memorizing. The modern geometry makes the study of definitions more interesting and educational. Definitions are illustrated by means of figures and exercises in order to emphasize the meanings of the terms defined and their uses. The artist-teacher guides the pupil to discover his own definitions by means of concrete illustrations, induction, and generalization. The definitions are made to grow out of the pupil's experience and activity.

3. *The Assumptions.* There is at present a better understanding of the foundations of geometry. A few decades ago teachers and textbook writers imagined that axioms were self-evident truths and that all the assumptions were explicitly stated as such. The work of mathematicians in recent years has dispelled these illusions. It is now seen that axioms are merely assumptions and that many assumptions are used without being explicitly formulated. There has been a decided trend to increase the number of assumptions so that Euclid's five axioms and five postulates have been increased to about thirty assumptions. Thus, the principle of rigor has been sacrificed for the educational principle of adapting the subject to the mind of the learner.

4. *The Syllabus.* The present century has witnessed a notable change in the syllabus of geometry. An analysis of the leading texts used in the last quarter of the nineteenth century shows that the number of basal propositions in plane geometry was over 160, whereas the leading texts at present average about 110 propositions. There seems to be a general agreement that a more careful study of a small number of propositions achieves better results in attaining the main objectives of geometry. This reduction in the syllabus has been brought about in several ways. About a dozen propositions that had been proved formally in the texts of the nineteenth century are now treated informally or postulated in the interest of simplicity. The former theorems on proportion are now treated algebraically without being designated as propositions. Then several former propositions are now included among the exercises. The incommensurable cases have been eliminated from most of the modern texts. Their meaning is made clear by means of illustrations and

the pupils are frankly told that the proof is incomplete because it is difficult at this stage of progress. One of the chief advantages of the briefer syllabus is that it allows more time for original work and discovery.

5. *Incomplete Proof.* One of the important changes in geometry is with regard to the nature and form of the proofs of the basal propositions. The texts of the nineteenth century presented the proof of each basal proposition in full, giving all the statements of a proof and the reason for every statement. The complete proof was considered essential for providing the pupil with standard forms exhibiting the best and most clearly stated demonstrations that geometry contains. Memorizing these proofs was made the main work in geometry. Critics of the complete proof during the first decade of the present century argued that pupils had committed these full proofs to memory and yet were unable to apply them in the solution of originals. They said that one of the chief values of the study of geometry is to develop the power of sound reasoning; and no amount of memorizing facts and proofs that another has discovered will give one the power to find facts and prove them for himself.

Recent texts aim to prevent the mechanical memorizing of proofs by introducing the incomplete or suggestive proof. According to this plan the majority of the basal propositions are proved in full, thus conserving the advantages of keeping model proofs before the pupil throughout the course. In many theorems, however, the statements of the proofs are given, but the reasons are purposely omitted with only a question mark or query indicating that the pupil should supply them. In many instances, where the proposition is simple, the proof is left entirely to the pupil. Recent texts leave about one third of the proofs incomplete. The incomplete proof is an incentive to real thinking and a better understanding by members of the class. It helps to create in the pupil the spirit of discovery, and to give him training in exactly the kind of thinking which he needs in attacking originals.

6. *Style.* Another significant development is in the style and form of the proof. The early texts presented the proof in the essay type. They used the formal style of a scientist addressing adults and scholars in the language of rigorous logic. Modern texts use the more natural conversational language. The essay form has been replaced by concise statements in which symbols and detached statements are used in the interest of clearness and distinctness. The statements and reasons of a proof are now presented in numbered steps and parallel columns. This parallel column arrangement seems to emphasize more strongly the necessity of giving a reason for each statement made, and it saves time when the pupils are following a proof or when the teacher is inspecting and correcting written work. Also, the unit page, which was introduced before the end of the nineteenth century, has now become universal. As a result of all these improvements the geometry text has become not only a work of art and beauty, but also an excellent instrument of instruction.

7. *The Originals.* Probably the most striking change in the teaching of geometry is the new emphasis on originals. When geometry became a high-school subject about sixty years ago originals were almost unknown. Thirty years ago exercises had become a permanent feature of the geometry text. They were massed in groups at the end of the various books, and they were not graded according to difficulty. There still existed a general feeling that they were formidable and far beyond the ability of pupils to be included in the course. Only the exceptional teacher understood

their true function and made them an important part of the course.

Since the beginning of the present century, there has been a decided tendency to increase the number of exercises in the geometry text. The average number of exercises in six of the leading texts of the nineteenth century is 265, while the average number for six of the leading texts at present is about 1,600, with one text containing as many as 2,156 exercises. The exercises are no longer massed at the end of the books, but are well distributed throughout each book. They are also graded according to difficulty, the first exercises, being simple, involving only one step. When a pupil finds that he can readily solve a simple exercise, he experiences the joy of achievement and is willing to try a harder one. It is better for a beginner to solve half a dozen simple exercises readily and develop self-confidence than to spend an equal time on a difficult exercise and be discouraged by failure. The modern selection and arrangement of exercises takes care of individual differences and encourages every pupil to attain a level of achievement according to his ability.

Some of the latest geometry texts have introduced new-type exercises such as completion exercises, true-false exercises, matching exercises, and exercises requiring the choice of correct or best answers. These exercises are objective, time saving, and interesting. If they are used not to supplant but to supplement the substantial originals in the regular text, they constitute a real improvement in the teaching of geometry.

Thus, while thirty years ago almost all the emphasis was on the mastery of the proofs of the basal propositions with very little attention given to exercises, today the chief emphasis is on exercises with the basal propositions considered as tools to be used in the more important work on originals. This change is without exaggeration nothing less than revolutionary. No other subject in the high-school curriculum has shown a greater capacity for adaptation to the demands of modern education than the course in geometry.

8. *Applications*. When geometry was a college subject, it was taught as an abstract science. After it was handed down to the high school to be studied by less mature minds, it continued to be taught in the same way. Even the exercises were entirely abstract, purely geometrical. The modern demand that a school subject should function in the life of the individual led the teachers of geometry to endeavor to vitalize their subject by teaching it in relation to its actual uses in the practical world. Modern texts point out the practical applications of geometry in art and decorative design, in architecture and industry, in field measurements and the making of maps, in engineering and science. Another means of vitalizing geometry is the introduction of applied problems that are encountered in the various practical fields of human activity. Every successful text published during the past two decades provides numerous applied problems to let the pupil see how the principles and applications of geometry underlie our modern civilization. Some recent texts offer more than two hundred practical problems.

9. *The Function Concept*. The function concept has been increasingly emphasized in recent years. In answer to a questionnaire with regard to the objectives of geometry, a number of teachers stated that training in functional thinking should be the supreme aim of the teaching of geometry. Every unit of geometry offers opportunity for training in functional thinking. The idea of functionality is bound to play a fundamental rôle in the study of geometry, not only because it will give the pupil a better mastery of geometry and prepare him for a better understanding of science

and of advanced mathematics, but also because the habit of thinking about the connections that exist between related quantities will make him better able to think more clearly about the actual relations which he meets in the activities of life.

10. *Generalization.* Another idea which is receiving greater emphasis in recent teaching is that of generalization. The cultivation of the power of generalization is considered by psychologists as the most important achievement in the student's education. Geometry lends itself to the process of generalization. There is an increasing number of teachers who believe that work in geometry should consist largely not of statements of theorems to be proved, but of specific problems to be investigated with the purpose of discovering some property that can be generalized and then lifted up by a logical proof from the realm of probability to the realm of scientific certainty. An interesting form of generalization is the grouping of several related theorems as special cases and the formulation of a general statement which includes them all. Training in this phase of generalization will help the pupil to understand more clearly the all-important principle of continuity.

11. *Reasoning in Geometry and Life.* During the past two decades, teachers of geometry have paid special attention to the applications of the facts and principles of geometry in the activities of life. Leading teachers are now emphasizing certain geometric processes of thought which are of the utmost importance to every one in actual life. They seek to teach geometry in such a way as to make it a model for all life thinking. More and more emphasis is being placed on the meaning and method of proof. One prominent teacher calls his course in geometry a course in straight thinking. Geometry insists upon logical reasoning as against instinct, appearance, or prejudice. Such processes of thinking as the indirect proof, analysis, reasoning by exclusion, deductions, are receiving special attention in the teaching of geometry. The need of avoiding hasty generalization or reasoning from insufficient data; of avoiding reasoning in a circle, or begging the question; of avoiding vague definitions and vague arguments is equally great in geometry and in life. The ability to grasp a situation, to seize on the few central facts or ideas, and marshal all the subsidiary facts and ideas around them can be cultivated by means of proper geometric training.

12. *Correlation.* One of the most significant developments during the last thirty years is the correlation movement which seeks to bring the various branches of mathematics into closer touch with each other and with related subjects. There is a wider use of algebra in the geometry course. The essay type of proof which prevailed half a century ago has been abandoned, and algebraic symbolism has been introduced in the proof to make it more concise and clear. Some theorems, such as the Pythagorean theorem, are proved simply and elegantly by methods involving algebra. The use of algebra often enables one to work certain exercises readily. There is also a closer correlation between geometry and trigonometry. Nearly all the modern texts include a brief treatment of the solution of right triangles by means of the sine, cosine, and tangent in connection with the unit on similarity. This makes possible the introduction of many interesting practical problems in field measurements. Teachers have also sought to bring a closer relation between geometry and physics, especially by means of exercises involving practical applications. While the correlation movement has had a marked beneficial influence on the teaching of geometry, the great body of teachers is convinced after considerable experimentation that demonstrative geom-

etry will not blend with other subjects to any great extent without losing its chief educational value.

13. *Purpose of Geometry.* Perhaps the most important change in the teaching of geometry is in the real purpose of the course. The purpose of the teaching of geometry during the last quarter of the nineteenth century was to prepare students for college, to discipline the mind, and to make mathematicians. The purpose today is to prepare boys and girls for life and to train intelligent American citizens. Geometry is today making a greater contribution because it is being adapted to the pupil. Consideration of his interests, capacities, and social needs are the primary factors in determining the objectives of the course and the new technique of instruction. It is no longer sufficient to tell pupils to study geometry because it trains the mind and because the board of education requires it. The alert teacher will pay attention to motivation. He will make geometry so interesting that pupils will take pleasure in studying it. When pupils learn the purpose of a course or of a unit, they are more interested in it, superior results are obtained, and their study becomes a combination of pleasure and work.

14. *Preparation of Teachers.* During the past thirty years there has been a steady improvement in the personnel of the teachers of geometry, a general raising of standards, and a better professional preparation. Indeed, the greater part of the body of modern teachers has been alert to the changing order, responsive to the spirit of the age, and active in contributing to the gradual evolution in the teaching of geometry. The nature of the experimental work that is being done in the classroom, and the excellent reports of the committees of the mathematical associations are evidences of the progressive character of the teachers of geometry. Leading teachers are constantly seeking to make new improvements that geometry may be in the forefront of every forward movement. Not satisfied with the impressive record of past progress, they are alert to find new problems for discussion and solution. For a decade they have been advocating the teaching of a unit of demonstrative geometry in the ninth grade, the purpose of which is to show pupils what demonstration means and to bridge the existing gap between intuitive and demonstrative geometry. There is also at present a movement to bring about the introduction of a course including the essentials of plane and solid geometry in one year's work in the place of the present course in plane geometry. The chief object of this plan is to secure for a larger number of pupils some of the important values of the study of solid geometry without sacrificing any of the essential values of the present course in plane geometry. The writer believes that the next step forward should be the introduction of concrete geometry into the mathematical work of the elementary school.

The foregoing discussion should eradicate the all too common feeling that geometry is static. Quite the contrary has been shown to be true. Probably no other subject found in the high-school curriculum has undergone so marked a change in its purpose, its content, or its method. Let the teacher of geometry realize that his subject is alive and growing, let him take active interest in the new developments, and his teaching will be a joy to him and an inspiration to his pupils.

A MATHEMATICAL RECREATION

M. A. SAUERBREI

EDITOR'S NOTE: *Mr. Sauerbrei is in charge of mathematics in the progressive Country Day School at Rye, New York. In addition to meeting formal entrance requirements in mathematics, Mr. Sauerbrei is stimulating individual creative work.*

J. R. C.

A wave of excitement runs through the class as the next player takes her place at the blackboard. The bases are full, there are two out, and the score is tied. The next example may decide the game. Quickly it is chosen from one of the several available textbooks and the work begins. Each member of the class has sought a spot which affords an unobstructed view of the blackboard, and watches the work intently. Each hopes that she will be the first to discover an error and thus aid her side. Slowly, carefully, the player works until a sudden shout announces the fact that the problem has been successfully completed and that the winning run has been scored. Our best athletes have never received more enthusiastic praise than has been bestowed several times upon pupils who, in the classroom, have thus performed brilliantly and have saved their team from defeat.

This game, algebraic baseball, is the result of an attempt to accelerate classroom procedure, to maintain interest, and to stimulate each pupil to maximum effort during the part of the term given over to review or drill. The school should aim to get constantly the best effort from each individual pupil. In the daily work, maximum and minimum assignments provide for the varying needs of pupils of differing abilities. In the review periods, however, where the class recites as a unit, the situation becomes more complex and it is much more difficult to devise a procedure which meets individual needs and furnishes all with an incentive to work.

At the completion of the work in factoring, or of any other division of the year's work, the standard textbooks contain an exercise of from sixty to one hundred miscellaneous review examples. The far too common practice is to assign the same examples to all members of the group and to take two or three days to complete the exercise. Obviously, this is unjust, for while this may be a valuable exercise for the slower members of the group, there are not over twenty-five per cent of these examples that the upper half of the class profits by doing. To do this other seventy-five per cent means that a great deal of time is wasted by the pupils who are most capable of using this time to an advantage in some other way. In providing all these pupils an immediate incentive to excellence, "baseball" serves admirably.

The procedure is quite different from that of the usual review exercise. The assignment reads, "To understand and be able to do any of the factoring examples to be found in the review exercises in the different algebra books on my desk. These examples will be used in the game tomorrow." All day long pupils come in groups and use one book after another. Each pupil understands that it will pay her best to direct her main effort upon that particular type in which she is weak and to waste no time on the types she has already mastered. Even the brightest members of the class find many examples worth their while.

If a second or a third day of review is needed, the pupils themselves can make up the examples to be used. As the success of their team depends upon the examples which they bring into class being of an appropriate type, considerable thought is given

to them. Every morning, after this kind of an assignment has been made, a number of pupils come eager to show how they have combined several processes into one example and have so rearranged them that they feel certain their opponents can never find the solution. It is surprising how quickly the pupils discover the most common errors and make up examples involving the probability of such errors. In so doing they are unconsciously protecting themselves against making these same errors.

The rules of the game are simple. The class is divided into two teams, a batting order is made out and a diamond, to be used for scoring, is drawn on the board. One side is at "bat," the other side in the "field." One "batter" at a time takes her place at the board and is given an example by a member of the opposing team. If the example is completed correctly, it counts as a safe "hit" and a cross is placed at first base to show that a "runner" has reached there safely. Each successive "hit" advances the "runners" one base. Therefore, obviously, there must be four "hits" before there are three "outs" in order to score a single "run." Each "hit," after the bases are full, scores another "run." The pupils watch the work constantly and if an error is discovered they raise their hands immediately. If the side in the field is the first to discover the mistake, the "batter" is "out," but if a member of the side at "bat" is first, the "batter" may correct the error and continue. If, however, one of the side at "bat" raises her hand to indicate an error when there is none, the "batter" is "out," and if a member of the opposing team makes the same mistake, the "batter" is credited with a "walk" and does not need to complete the problem. The members of each team must follow the work closely and be able to make correct decisions quickly in order to score for their side. The teacher simply decides which hand is raised first and thus which side is entitled to the advantage thus earned.

The writer does not claim that this plan is a panacea for all classroom evils. But it is one device for stimulating every pupil to master each process covered by a review exercise and for creating a classroom situation in which there is enthusiasm, concentration, and one hundred per cent participation. As such, this game has proven to have much merit.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

JOHN P. EVERETT AND WILLIAM D. REEVE

EDITOR'S NOTE: *Mr. Everett is head of the department of mathematics of the State Teachers College at Kalamazoo, Michigan, and President of the National Council of Teachers of Mathematics. His* Fundamental Skills of Algebra *is widely and favorably known. Few persons are doing more than Mr. Everett for the improvement of instruction in mathematics. Mr. Reeve is professor of mathematics at Teachers College. Under his leadership, the Mathematics Section of the New York Society for the Study of Education has become a large and powerful organization of teachers of mathematics. Mr. Reeve is editor of both* The Mathematics Teacher *and the Yearbooks of the National Council of Teachers of Mathematics.*
J. R. C.

Legal enactments are necessary to the maintenance and administration of schools. Throughout the world, education is looked upon as being a matter of public concern to such an extent that certain standards for schools and teachers and the granting of certificates and degrees are defined in terms of law.

There are, however, many attributes of the educational process which no legislature can control. Enthusiasm and respect for one's work; the ability and disposition to

adapt content material and curricula to changed and changing economic and social conditions; research for the purpose of retaining and evaluating former, and sometimes ancient, learned discoveries, as well as research for the purpose of developing newer ideas and points of view; a desire to avail one's self of the means of accomplishment which others in the same general line of work have proved effective; and a desire to contribute to the standing and the genuineness of one's profession—all these are as necessary to schools as are textbooks or school buildings or taxes or diplomas, yet none of them may be created or more than mildly stimulated by the laws of a nation or of its divisions.

Such qualities and attributes of the educational process as have been mentioned exist, thrive, and find their best revelation in an atmosphere of spontaneous expression. The National Council of Teachers of Mathematics is a voluntary organization of teachers of mathematics having for its purpose the promotion and stimulation of that kind of a spirit of teaching which strives to do a better job than any appointment to a position or any contract is able to enforce.

To accomplish its purpose the National Council operates chiefly through three divisions of its organization; namely, *The Mathematics Teacher*, the Yearbook, and the annual meeting of its members and board of directors.

The Mathematics Teacher is a periodical, published monthly, except during the vacation season, and mailed to each of the six thousand members of the Council. A printed index by titles and subjects covering the last nine years of *The Mathematics Teacher* comprises sixty-eight pages and lists over three hundred authors. The names of contributors include those of the most distinguished classroom teachers, textbook writers, and administrators to be found in this and several foreign countries. Sources of material and quotations range from the Rhind Papyrus of 1700 B. C. to the dissertation of the most recent doctor of philosophy. Subject matter runs the scale from arithmetic problems to advanced calculus and vector analysis. Methods are discussed from those of "subtraction" to those of "curriculum making."

Into the hands of teachers scattered throughout every State, *The Mathematics Teacher* carries its content and its inspiration, breathing the interest of today, but emphasizing the wisdom of all time. No teacher, probably, finds all of its pages equally valuable, but there is always something to apply directly to the problems of the classroom, and the presentation of ideas beyond the sphere of service of any individual serves to keep alive the thought and progress of the times.

Since its beginning in 1908 *The Mathematics Teacher* has contained numerous and varied articles on the teaching of mathematics. The following list of titles and the accompanying frequencies will give some idea of the range of topics treated:

Algebra	40
Arithmetic	20
Calculus in high school	4
College entrance and regents	10
College mathematics	7
Curriculum	61
Experimental work	34
Function concept	9
General mathematics	24
Geometry	48
Higher mathematics	3
Historical	27
Mathematics in foreign countries	8
Methods	110
Recreations	23
Reports of associations and committees	36

The Council has published five yearbooks which include articles of greater length and

of more rigorous scientific authority than *The Mathematics Teacher* in its capacity as a monthly periodical may be expected to contain. The general topics of the yearbooks in order are: "A general survey of progress in the last twenty-five years," "Curriculum problems in teaching mathematics," "Selected topics in the teaching of mathematics," "Significant changes and trends in the teaching of mathematics throughout the world since 1910," and "The teaching of geometry."

The sixth Yearbook will be published in February 1931, and is entitled "Mathematics and Modern Life." This book will contain the contributions of several eminent physical and social scientists, and will indicate the bearings which these authorities regard mathematics as having on modern thought in their respective fields.

Some of the contributions of the yearbooks are especially noteworthy as affecting the trends of instruction in mathematics. Among them may be mentioned "The Foundations of Mathematics," delivered as a presidential address by E. H. Moore before the ninth annual meeting of the American Mathematical Society. Publication of this article in the first Yearbook has brought the address of Professor Moore to the attention of thousands of teachers. The second Yearbook, with its presentation of curriculum problems, has served to furnish a working background and a unifying congeries of ideas for schools and teachers who are trying to put into effect some of the recommendations of the National Committee on the Reorganization of Mathematics. A distinct contribution to clarity of thought and vision is to be found in C. H. Judd's article in the third Yearbook on "The Fallacy of Treating School Subjects as *Tool Subjects.*"

The brief references that have been made to the yearbooks may be taken as typical of the strength and character of their offerings. The books are to be found in all libraries which further the education of teachers.

Affiliation with the National Council

Any club or association wishing to affiliate with the National Council should first pass a resolution stating that the group desires to be officially affiliated with the National Council and authorizing the officers of the group to transmit the request to the Secretary. With this request, a list of members should be forwarded with the names of those already members of the National Council clearly indicated; or, in case of the larger organizations, a statement of the approximate number of members and the number belonging to the National Council.

The Executive Committee of the National Council will act promptly upon all such applications and will notify the petitioners accordingly.

Secondly, the officers of the group should encourage members so far as possible to join the National Council and to remit the $2.00 which pays for membership for one year and which includes subscription to the official journal, *The Mathematics Teacher.* Thirdly, a list of the officers of the group should be filed with the Secretary of the National Council. This list should be kept up-to-date at all times.

In a city, State, or section, any number of interested persons may organize and submit a petition to become affiliated. After the members of the group decide upon the territorial limits and elect their officers the procedure is similar to that stated above. The national officers are ready at all times to coöperate in the organization of new local associations. In sections not densely populated, it may be advisable to have bi-State affiliated groups. Such a group was organized in 1927 in Louisiana and Mississippi. It meets in connection with the Louisiana-Mississippi Section of the Mathematical Association of America. Such regional or

State groups may be subdivided for more frequent sectional meetings than are possible for the larger groups.

An existing organization need not change its name or lose its identity in order to become affiliated with the National Council. It should keep its name and merely append to the name a descriptive title.

The annual meetings afford a forum for reports and discussions of the main topics of thought and action in contemporary teaching of mathematics. At the next meeting, which is to be held in Detroit, February 20 and 21, 1931, Professor Dunham Jackson, of the Unversity of Minnesota, and Mr. C. M. Austin, of Oak Park High School, Illinois, will furnish extensive reports of progress that has been made in the reorganization of courses in high-school geometry. Mrs. Suzanne K. Langer, of Cambridge, Massachusetts, will deliver an address upon the subject, "Algebra, the Special Training Ground of the Reason." The method and the manner in which mathematics widens the horizon of the mind will be discussed by Professors Schorling and Anning of the University of Michigan. Other speakers equally qualified to interpret the significant trends of the day will appear upon the program, which will close with an address by Professor E. R. Hedrick, of the University of California at Los Angeles, upon the subject, " Training Teachers with Especial Reference to the Relation of Mathematics to Modern Thought." Professor Hedrick is chairman of the American Committee of the International Commission on the Teaching of Mathematics.

The editor-in-chief of both *The Mathematics Teacher* and of The Yearbook is Professor W. D. Reeve, of Teachers College, Columbia University, with offices at 525 West 120th Street, New York City.

BOOK REVIEWS

The Teaching of Secondary Mathematics, by J. O. Hassler and R. R. Smith. New York: The Macmillan Company, 1930, 405 pages.

This book should appeal to the young teacher who has not had the time to formulate a working philosophy of his own. It should also find a place to function in teacher-training classes. It may be of passing interest to the teacher of more experience but the reviewer does not care to recommend without some reservations. The experienced teacher would prefer to find some attempt made to evaluate the outstanding experiments now going on in the classroom. The references at the end of every chapter are good but they would have been better if they had been annotated.

The first part of the text presents not only a brief historical account of the principal topics of mathematics but implies the possibility of a philosophy of mathematics that is within the reach of the secondary-school teacher. The second part of the book contains many cut-and-dried recipes for the teaching of mathematics in the secondary schools. There is a bibliography and an index.

M. R. M.

Solid Geometry, by F. E. Seymour. New York: American Book Company, 1930, 231 pages.

If the reader liked the plane geometry text by Mr. Seymour, he will want to supplement it with this text. The training in logical demonstration is not neglected but it is not treated as of the first importance. The emphasis is placed on the training in spatial imagination and in solving problems in mensuration. Can such a course compete with the use of models in the junior high and the evaluation of mensuration formulas throughout the algebra? Mr. Seymour's text is one of the best, but something better is needed to retain solid geometry as a separate unit in our high schools.

M. R. M.

Solid Geometry, by Joseph A. Nyberg. New York: American Book Company, 1930, 456 pages.

In this text the work in demonstrative geometry is rigorous enough to command the admiration of that unfortunate one who failed to develop a true appreciation in the plane geometry course. The first proofs are given fully, but later only the sug-

gestions as to general procedure are offered. The use of formulas in developing the functional dependence of variables is well handled. Provision has been made for a differentiated course by the use of light-face type for the supplementary work. The text is a worthy supplement to the plane geometry text by the same author.

M. R. M.

Three Comedies of Shakespeare. New York: Harcourt, Brace & Co., 1929, vi+409 pages.

A beautiful edition of "Merchant of Venice," "Tempest," and "As You Like It" in one volume. The type is large and the full-page illustrations add immensely to its attractiveness. It should make an excellent gift book.

M. G. W.

Abe Lincoln Grows Up, by Carl Sandburg. New York: Harcourt, Brace & Co., 1928, 222 pages.

This book is a reprint of the first twenty-seven chapters of *Abraham Lincoln, the Prairie Year.* It covers Lincoln's birth, boyhood, years at Knot Creek farm, and his life until he leaves home at nineteen. The book is beautifully illustrated by James Daugherty. It would make an excellent gift for any boy or girl.

M. G. W.

Supervising Extra-Curricular Activities, by Paul W. Terry. New York: McGraw-Hill Book Company, xi+417 pages.

Student activities in the American public schools are demanding increasingly greater attention. As a result, many questions relative to the possibilities of these various activities and their proper supervision confront practically every modern secondary school. The author of this new volume endeavors to cast some light on the merits as well as on the dangers of various student activities and seeks to indicate how, in the light of the most progressive practice, such activities may be handled by the school. The work falls into four divisions. The first division presents a brief historical background in which the author outlines the "growth trends of the past" and "the extent to which student activities in the past have been shaped by the social conditions and educational theories which formed their environment." In the second and third divisions various types of student activities are described and their advantages and disadvantages are pointed out. Such activities as student participation in school government, honor societies, school publications, forensics, dramatics, athletics, clubs, orchestras, and fraternities and

sororities come under consideration. In Part IV, administrative and supervisory programs, which are generally encountered in connection with the different student organizations, receive attention. Such problems as the extent and variety of student participation, the relation of intelligence to participation, the control of participation, organizing and financing activities, and the nature and extent of supervision receive practical consideration.

The book should be of interest and help to those administrators and supervisors who are confronted with problems pertaining to proper organization and effective supervision of student activities in secondary schools. Each chapter is provided with a rather extensive bibliography which should prove of value to those who desire further investigation of special topics. C. L. W.

Enriched Teaching of Mathematics in the High School, by Woodring and Sanford. New York: Bureau of Publications, Teachers College, Columbia University, 1928, 128 pages.

The purpose of this book is explained in the subtitle. A source book for teachers of mathematics, listing chiefly free and low-cost illustrative and supplementary materials. It is a book which should be on every mathematics teacher's desk. To show the wide range of topics covered, a few are listed at random, all dealing with material on mathematics, of course: graphs; the slide rule; recreation and puzzles; assemblies; motion pictures; plays; diagnostic tests; appreciation of mathematics; exhibits; history of mathematics; and many others. The book is one of a series called the "Enriched Teaching Series" of which volumes on English, science, and commercial subjects have already been published. M. G. W.

Problems in Architectural Drawing, by Franklin G. Elwood. Peoria: The Manual Arts Press, 1930, 88 pages.

A functional treatment of the subject architectural drawing setting forth the problems involved in wood, steel, masonry, and reinforced concrete construction. The fifteen tables listed towards the end of the book give data on the strength of various building materials, which data are very necessary in the solution of construction problems.

For the purpose of applying the content of the book the author presents nine problems ranging from a filling station to a city apartment house. Many excellently made plates and diagrams add to the understanding and value of the book.

E. R. G.

Using English, by Lucy H. Chapman. New York: Harcourt, Brace & Co., 1929, xi+498 pages.

A successful young man recently said in a talk to the students of his former high school, "What the world demands today of young men and women is the ability to communicate with their fellows and technical knowledge." A book especially created to meet that first demand is *Using English,* as a list of its contents shows. Part I teaches how to tell stories; write friendly, social, and business letters; give oral and written directions, explanations, and reports; make clear recitations; conduct meetings; and perform office duties including handling correspondence, writing telegrams, meeting callers, and answering the telephone. Part II contains grammar, grammar, and more grammar. Important points a teacher would note are: each topic is thoroughly discussed in one place; provision is made for ability grouping; there is a very large number of drills of all kinds.

A most unusual feature is a numerous number of excellent cartoons; they are not only highly entertaining, but they serve to make clear many grammatical pitfalls in an unforgettable manner.

The book was written for the ninth and tenth grades, but it could well serve as a handbook for any one who wanted an interesting and authoritative book on the art of communicating by tongue or by pen with his fellows. M. G. W.

Speaking in Public, by Arleigh B. Williamson. New York: Prentice-Hall, Inc., 1929, xvi+412 pages.

Professor Williamson has given us an exhaustive study of the outstanding aspects of the problem that confronts the student who aims to become an effective speaker in public. The topics are presented from the point of view of the student but with the authority of an accomplished speaker who has analyzed the steps by which he has attained mastery of his art. The text concerns itself not only with the delivery of speeches, but with their composition, and emphasis is laid on gathering material, planning, and adapting the substance to the occasion. The chapters on "Oral Delivery" and "Getting Rid of Vocal Monotony" seem to this reviewer to be particularly worth the attention of students. Much that is in the book should be of distinct value to teachers of all subjects.

A. D. W.

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